

Online Appendix to “General Equilibrium Effects in Space: Theory and Measurement”

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June 2020

A Online Appendix: Empirical Application

This appendix presents additional empirical results that complement the baseline estimates presented in Section 6. Section A.1 complements the estimates of the simple extension of ADH’s specification in Section 6.3. Section A.2 presents extensions of the baseline estimates of the reduced-form elasticities in Section 6.4.2. Section A.3 complements the results on the baseline predicted effects shown in Section 6.5.

A.1 Simple Extension of ADH

A.1.1 Robustness of Baseline Estimates in Table 1

This section investigates the robustness of the baseline estimates in Table 1. Given the non-significant responses to consumption cost shifts, we focus on our preferred specification in column (5) where we include only the indirect effects of revenue shifts.¹

Table A.1 investigates the importance of the three sets of controls used in our baseline specification. Column (1) only includes the baseline controls in ADH: period dummies, college-educated population share in 1990, foreign-born population share in 1990, employment share of women in 1990, employment share in routine occupations in 1990, average offshorability in 1990, and Census division dummies. We can see that, while the effect of revenue shifts is negative and significant on wages, it is not significant on employment. This is consistent with the results in ADH who find a significant negative effect of Chinese import exposure only on the number of employed individuals in the manufacturing sector. In their baseline specification, the impact of shock exposure on the total number of employed individuals in the CZ is negative but it is not statistically significant at usual levels – see also Bloom et al. (2019) for a discussion about this point.

In column (2), we add controls for the total exposure of the CZ to shocks in the manufacturing sector, both directly and indirectly. In particular, we include the following controls: manufacturing employment share ($\sum_s y_{i,s}^{t_0}$), manufacturing spending share ($\sum_s \xi_{i,s}^{t_0}$), indirect exposure to manufacturing employment

¹For the specification in column (6), results are typically similar, but standard errors are larger. This is because the correlation of the indirect exposure to revenue and consumption shifts is high, 0.53.

share ($\sum_{j \neq i} z_{ij} \sum_s y_{i,s}^{t_0}$) and manufacturing spending share ($\sum_{j \neq i} z_{ij} \sum_s \xi_{i,s}^{t_0}$). This set of controls absorbs the impact of the secular decline in manufacturing employment. Results show that estimated coefficients are very similar to those in column (1).

Finally, in column (3), we follow Greenland et al. (2019) by controlling for the 10-year lagged growth of the population that is 15-34 and 35-64 years old. These controls capture potential confounding effects of slow moving trends in population growth. This control set has almost no effect on the estimated coefficients for wage responses. However, for the employment responses, controlling for population trends yields higher point estimates and smaller standard errors. This is consistent with the evidence in Greenland et al. (2019) that obtain more precise population responses to trade shocks when controlling for past population growth.

Table A.2 investigates how our baseline results change when we modify the function z_{ij} specifying the indirect effects. In all columns, we normalize $\sum_{i \neq j} z_{ij} IPW_j^t$ so that the reported coefficient is the impact of changing indirect exposure by one standard deviation. In columns (1)–(4), we maintain the same functional form for z_{ij} , but modify the value of δ . For all specifications, the indirect effects are negative and statistically significant, but the point estimates are higher when δ is lower. In column (5), we show that point estimates are slightly lower when we ignore the CZ size in the computation of z_{ij} . Finally, in column (6), we specify z_{ij} to only assign positive weight to other CZs in the same state. In this case, the point estimates are slightly higher, reflecting again the fact indirect effects decay with distance.

Table A.3 reports results obtained with two alternative specifications. Columns (1) and (3) replicate the baseline estimates in columns (5) of Table 1. In column (2), we follow ADH by weighting CZs by their 1990 population. Note that this weighting scheme is not consistent with our theory in which the unit of analysis is a market. Results suggest similar estimates of the indirect effect for both employment and wages. However, conditional on the indirect exposure, the direct effects are not significant. Notice that, in this case, responses to stronger declines in consumption prices are negative. So, the consumption exposure may be partially capturing the revenue exposure of large CZs. This further confirms that there is no evidence of a positive impact of lower prices on employment. Columns (3) and (6) report standard errors computed with the inference procedure in Adão et al. (2019) that accounts for spatial correlation of the residuals. Although confidence intervals are wider, the indirect effects are still significant at 10%.

Table A.4 investigates how our estimates vary with the procedure to compute the spending shares $\xi_{i,s}^{t_0}$. In column (2), we show that all coefficients are similar when we ignore final spending in the computation of $\xi_{i,s}^{t_0}$. In this case, the cross-regional variation in IPC_j^t is entirely driven by intermediate spending shares. This suggests that local outcomes have only weak responses to import shocks that reduce the input prices for the industries in the CZ. One potential concern with our measure of the consumption shift is that, by including the own-industry spending, $\xi_{i,s}^{t_0}$ may capture part of the effect of Chinese import competition in the industry. To address this concern, column (3) shows that our estimates are similar when we compute $\xi_{i,s}^{t_0}$ ignoring input spending on the own sector.

Table A.1: Impact of the China Shock on Labor Market Outcomes across U.S. CZs, Alternative Control Sets

| | (1) | (2) | (3) |
|--|----------------------|----------------------|----------------------|
| Panel A: Change in avg. log weekly wage | | | |
| IPW_i^t | -0.531*** (0.138) | -0.362** (0.161) | -0.319** (0.152) |
| IPC_i^t | 0.146 (0.208) | -0.021 (0.210) | -0.043 (0.208) |
| $\sum_{j \neq i} z_{ij} IPW_j^t$ | -1.051*** (0.257) | -1.127*** (0.369) | -1.036*** (0.309) |
| R^2 | 0.518 | 0.522 | 0.536 |
| Panel B: Change in log of employment | | | |
| IPW_i^t | -0.301 (0.262) | -0.359 (0.220) | -0.468** (0.223) |
| IPC_i^t | 0.357 (0.476) | 0.506 (0.500) | 0.212 (0.419) |
| $\sum_{j \neq i} z_{ij} IPW_j^t$ | -0.100 (0.456) | -0.417 (0.509) | -1.330*** (0.345) |
| R^2 | 0.236 | 0.237 | 0.476 |
| Control set: | | | |
| Regional controls in ADH | Yes | Yes | Yes |
| Initial manufacturing exposure | No | Yes | Yes |
| Lagged population growth | No | No | Yes |

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. Indirect effects computed as in Table 1: $z_{ij} \equiv L_j^0 D_{ij}^{-\delta} / \sum_k L_k^0 D_{ik}^{-\delta}$ where $\delta = 5$, D_{ij} is the distance between CZs i and j , and L_j^0 is the population of CZ j in 1990. Regional controls in ADH: period dummies, college-educated population share in 1990, foreign-born population share in 1990, employment share of women in 1990, employment share in routine occupations in 1990, average offshorability in 1990, Census division dummies. Initial manufacturing exposure: CZ's share of employment and spending in manufacturing ($\sum_s y_{i,s}^{t_0}$ and $\sum_s \xi_{i,s}^{t_0}$), CZ's indirect exposure to manufacturing employment and manufacturing spending in other CZs ($\sum_{j \neq i} z_{ij} \sum_s y_{j,s}^{t_0}$ and $\sum_{j \neq i} z_{ij} \sum_s \xi_{j,s}^{t_0}$). Lagged population growth from Greenland et al. (2019): growth of population with 15-34 years old and 35-64 years old in the previous 10-year period. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table A.2: Impact of the China Shock on Labor Market Outcomes across U.S. CZs, Alternative Indirect Effect Specification

| | (1) | (2) | (3) | (4) | (5) | (6) |
|--|--|--|--|--|--|--|
| Panel A: Change in avg. log weekly wage | | | | | | |
| IPW_i^t | -0.551*** (0.147) | -0.525*** (0.139) | -0.494*** (0.131) | -0.513*** (0.134) | -0.489*** (0.128) | -0.568*** (0.145) |
| IPC_i^t | 0.188 (0.191) | 0.164 (0.193) | 0.126 (0.196) | 0.130 (0.195) | 0.112 (0.195) | 0.154 (0.192) |
| $\sum_{j \neq i} z_{ij} IPW_j^t$ | -6.928** (2.753) | -1.772** (0.815) | -0.903*** (0.292) | -0.726*** (0.195) | -0.731*** (0.196) | -1.690** (0.819) |
| R^2 | 0.534 | 0.530 | 0.532 | 0.530 | 0.531 | 0.534 |
| Panel B: Change in log of employment | | | | | | |
| IPW_i^t | -0.391 (0.235) | -0.360 (0.221) | -0.449** (0.213) | -0.470** (0.208) | -0.506** (0.207) | -0.534** (0.216) |
| IPC_i^t | 0.145 (0.417) | 0.138 (0.406) | 0.155 (0.403) | 0.116 (0.401) | 0.172 (0.400) | 0.223 (0.375) |
| $\sum_{j \neq i} z_{ij} IPW_j^t$ | -14.551*** (2.968) | -3.968*** (0.692) | -1.253*** (0.324) | -0.833*** (0.244) | -0.822*** (0.283) | -2.089** (0.904) |
| R^2 | 0.488 | 0.480 | 0.474 | 0.472 | 0.473 | 0.479 |
| Indirect Effect Specification: | | | | | | |
| Definition of z_{ij} | $\frac{L_j^0 D_{ij}^{-1}}{\sum_k L_k^0 D_{ik}^{-1}}$ | $\frac{L_j^0 D_{ij}^{-2}}{\sum_k L_k^0 D_{ik}^{-2}}$ | $\frac{L_j^0 D_{ij}^{-5}}{\sum_k L_k^0 D_{ik}^{-5}}$ | $\frac{L_j^0 D_{ij}^{-8}}{\sum_k L_k^0 D_{ik}^{-8}}$ | $\frac{D_{ij}^{-5}}{\sum_k D_{ik}^{-5}}$ | $\frac{L_j^0 St_{ij}}{\sum_k L_k^0 St_{ik}}$ |

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Indirect effects computed as specified in each column where D_{ij} is the distance between CZs i and j , L_j^0 is the population of CZ j in 1990, and St_{ij} is a dummy that equals one if CZs i and j belong to the same state. In all columns, we normalize $\sum_{j \neq i} z_{ij} IPW_j^t$ to have a standard deviation of one. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table A.3: Impact of the China Shock on Labor Market Outcomes across U.S. CZs, Alternative Specifications

| | 100 × Change in | | | | | |
|----------------------------------|----------------------|----------------------|--------------------|----------------------|----------------------|---------------------|
| | Avg. log weekly wage | | | Log of employment | | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| IPW_i^t | -0.319** (0.152) | -0.143 (0.219) | -0.319* (0.176) | -0.468** (0.223) | -0.735 (0.464) | -0.468** (0.225) |
| IPC_i^t | -0.043 (0.208) | -0.796** (0.328) | -0.043 (0.348) | 0.212 (0.419) | -0.662 (0.743) | 0.212 (0.453) |
| $\sum_{j \neq i} z_{ij} IPW_j^t$ | -1.036*** (0.309) | -1.036*** (0.304) | -1.036* (0.596) | -1.330*** (0.345) | -1.844*** (0.613) | -1.330* (0.743) |
| Specification: | | | | | | |
| Weight by population | No | Yes | No | No | Yes | No |
| Adão et al. (2019) SEs | No | No | Yes | No | No | Yes |
| State-clustered SEs: | Yes | Yes | No | Yes | Yes | No |

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Indirect effects computed as in Table 1: $z_{ij} \equiv L_j^0 D_{ij}^{-\delta} / \sum_k L_k^0 D_{ik}^{-\delta}$ where $\delta = 5$, D_{ij} is the distance between CZs i and j , and L_j^0 is the population of CZ j in 1990. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table A.4: Impact of the China Shock on Labor Market Outcomes across U.S. CZs, Alternative Spending Shares

| | (1) | (2) | (3) |
|--|----------------------|----------------------|----------------------|
| Panel A: Change in avg. log weekly wage | | | |
| IPW_i^t | -0.319** (0.152) | -0.365** (0.150) | -0.369** (0.148) |
| IPC_i^t | -0.043 (0.208) | 0.087 (0.104) | 0.128 (0.131) |
| $\sum_{j \neq i} z_{ij} IPW_j^t$ | -1.036*** (0.309) | -1.049*** (0.310) | -1.057*** (0.310) |
| R^2 | 0.536 | 0.536 | 0.537 |
| Panel B: Change in log of employment | | | |
| IPW_i^t | -0.468** (0.223) | -0.470** (0.217) | -0.449** (0.216) |
| IPC_i^t | 0.212 (0.419) | 0.110 (0.177) | 0.082 (0.211) |
| $\sum_{j \neq i} z_{ij} IPW_j^t$ | -1.330*** (0.345) | -1.328*** (0.344) | -1.327*** (0.346) |
| R^2 | 0.476 | 0.476 | 0.476 |
| Construction of IPC_i^t: | | | |
| Drop final spending | No | Yes | Yes |
| Drop own industry spending | No | No | Yes |

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Indirect effects computed as in Table 1: $z_{ij} \equiv L_j^0 D_{ij}^{-\delta} / \sum_k L_k^0 D_{ik}^{-\delta}$ where $\delta = 5$, D_{ij} is the distance between CZs i and j , and L_j^0 is the population of CZ j in 1990. In column (2), we compute $\xi_{i,s}$ ignoring final spending. In column (3), we compute $\xi_{i,s}$ ignoring also the own industry spending. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

A.1.2 Additional Results

This section provides additional results that complement our baseline estimates. We again focus on our preferred specification in column (5) of Table 1.

We start by investigating how other labor market outcomes respond directly and indirectly to regional shock exposure. Table A.5 shows that a negative revenue shock yields indirect negative effects on employment in both the manufacturing and the non-manufacturing sectors of other nearby CZs. This negative effect on employment translates into higher rates of non-participation and unemployment. Notice however that the impact on the unemployment rate is much smaller than the impact on the non-participation rate. In addition, column (2) reports a negative and significant effect of IPC_i^t on manufacturing employment. Table A.6 investigates the responses of wages in the manufacturing and non-manufacturing sectors. As in ADH, column (2) indicates that responses in manufacturing wages are not significant. The estimates in column (3) reveal that the direct and indirect effects of the shock on wages are concentrated in the non-manufacturing sector.

Finally, Table A.7 estimates how labor market outcomes responded to the removal of uncertainty on tariffs created by the NTR gap studied in Pierce and Schott (2016). To this end, we use an alternative shift to compute the shift-share variables measuring exposure in terms of revenue and consumption cost.

Instead of $\Delta M_s^{o,t}$, we use the simple average of the NTR gaps by 4-digit SIC industry. The results in Table A.7 show similar qualitative patterns to those in Table 1. First, both the direct and the indirect effects of higher revenue exposure are negative for wages and employment. Second, the impact of higher consumption exposure is non-significant for wages and employment – in this case, confidence intervals on the estimated coefficients are wider.

Table A.5: Impact of the China Shock on Employment Outcomes across U.S. CZs

| | Employed | | | Not employed | |
|---|----------------------|----------------------|----------------------|---------------------|---------------------|
| | Any sector (1) | Manuf. (2) | Non-Manuf. (3) | Unemp. (4) | NILF (5) |
| Panel A: Change in the share of working-age population by category | | | | | |
| IPW_i^t | -0.312*** (0.070) | -0.143*** (0.053) | -0.169*** (0.053) | 0.094*** (0.028) | 0.218*** (0.054) |
| IPC_i^t | -0.171 (0.131) | -0.189*** (0.057) | 0.018 (0.116) | 0.054 (0.045) | 0.118 (0.103) |
| $\sum_{j \neq i} z_{ij} IPW_j^t$ | -0.844*** (0.159) | -0.431*** (0.090) | -0.413*** (0.143) | 0.308*** (0.072) | 0.536*** (0.121) |
| R^2 | 0.321 | 0.556 | 0.236 | 0.313 | 0.288 |
| Panel B: Change in the log of working-age population by category | | | | | |
| IPW_i^t | -0.468** (0.223) | -1.021** (0.406) | -0.281 (0.209) | 1.966*** (0.667) | 0.861*** (0.248) |
| IPC_i^t | 0.212 (0.419) | 0.151 (0.648) | 0.444 (0.412) | 2.058* (1.030) | 0.917** (0.412) |
| $\sum_{j \neq i} z_{ij} IPW_j^t$ | -1.330*** (0.345) | -3.157*** (0.884) | -0.751** (0.343) | 7.158*** (1.885) | 2.037*** (0.730) |
| R^2 | 0.476 | 0.388 | 0.499 | 0.344 | 0.529 |

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Indirect effects computed as in Table 1: $z_{ij} \equiv L_j^0 D_{ij}^{-\delta} / \sum_k L_k^0 D_{ik}^{-\delta}$ where $\delta = 5$, D_{ij} is the distance between CZs i and j , and L_j^0 is the population of CZ j in 1990. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table A.6: Impact of the China Shock on Wages across U.S. CZs

| | 100 x Change in avg. log weekly wage | | |
|----------------------------------|--------------------------------------|-------------------|----------------------|
| | All (1) | Manuf. (2) | Non-Manuf. (3) |
| IPW_i^t | -0.319** (0.152) | 0.234 (0.216) | -0.427*** (0.147) |
| IPC_i^t | -0.043 (0.208) | 0.473 (0.500) | -0.009 (0.200) |
| $\sum_{j \neq i} z_{ij} IPW_j^t$ | -1.036*** (0.309) | -0.114 (0.523) | -1.128*** (0.306) |
| R^2 | 0.536 | 0.216 | 0.530 |

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Indirect effects computed as in Table 1: $z_{ij} \equiv L_j^0 D_{ij}^{-\delta} / \sum_k L_k^0 D_{ik}^{-\delta}$ where $\delta = 5$, D_{ij} is the distance between CZs i and j , and L_j^0 is the population of CZ j in 1990. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table A.7: Impact of the Removal of NTR Gaps on Labor Market Outcomes across U.S. CZs

| | (1) | (2) | (3) | (4) | (5) | (6) |
|--|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| Panel A: Change in avg. log weekly wage | | | | | | |
| $NTRPW_i^t$ | -2.852*** (0.415) | -2.653*** (0.470) | -1.565*** (0.368) | -2.425*** (0.412) | -1.410*** (0.388) | -1.410*** (0.388) |
| $NTRPC_i^t$ | | -1.419 (1.181) | | | -1.156 (1.193) | -1.214 (1.099) |
| $\sum_{j \neq i} z_{ij} NTRPW_j^t$ | | | -2.439*** (0.526) | | -2.425*** (0.529) | -2.488*** (0.654) |
| $\sum_{j \neq i} z_{ij} NTRPC_j^t$ | | | | -5.278** (2.467) | | 0.502 (2.477) |
| R^2 | 0.574 | 0.574 | 0.589 | 0.578 | 0.589 | 0.589 |
| Panel B: Change in log of employment | | | | | | |
| $NTRPW_i^t$ | -3.281*** (0.592) | -3.525*** (0.725) | -1.452** (0.573) | -3.031*** (0.648) | -1.735** (0.673) | -1.728** (0.667) |
| $NTRPC_i^t$ | | 1.743 (2.002) | | | 2.120 (1.928) | 1.451 (1.876) |
| $\sum_{j \neq i} z_{ij} NTRPW_j^t$ | | | -3.466*** (0.721) | | -3.492*** (0.719) | -4.227*** (0.672) |
| $\sum_{j \neq i} z_{ij} NTRPC_j^t$ | | | | -3.084 (2.564) | | 5.850* (3.086) |
| R^2 | 0.493 | 0.493 | 0.504 | 0.493 | 0.504 | 0.505 |

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Indirect effects computed as in Table 1: $z_{ij} \equiv L_j^0 D_{ij}^{-\delta} / \sum_k L_k^0 D_{ik}^{-\delta}$ where $\delta = 5$, D_{ij} is the distance between CZs i and j , and L_j^0 is the population of CZ j in 1990. We compute regressors using the shift-share definitions in Section 6.3 where the sector-level shifter is the NTR tariff gap in Pierce and Schott (2016) (instead of ADH's per-worker Chinese $\Delta M_s^{o,t}$). Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

A.2 Reduced-Form Elasticities to Revenue and Consumption Shifts

A.2.1 Robustness

This section investigates the robustness of our baseline estimates of the reduced-form elasticities to the assumptions embedded in the parametrization of spatial links described in Section 5.2. Table A.8 presents the estimates of the structural parameters using different specifications of spatial links. Panel A reports the baseline parameters in Table 3. In Panel B, we report the estimates obtained using an alternative specification of spatial links that allows for trade imbalances in each CZ. In this case, we use the reduced-form elasticities defined in Online Appendix C.1 and trade matrix implied by our imputation before adjusting market-level income to guarantee balanced trade everywhere. In Panel C, we use a different normalization of labor supply. Rather than using the world’s average wage, we use the national price index: in the labor supply specification (26), we impose $b_j^w = 0$ and $b_j^p = Y_j / \sum_{j \in C(j)} Y_o$ with $C(j)$ denoting the country of region j . Lastly, in Panel D, we calibrate the spatial elasticity of labor supply using a value similar to the one in Caliendo et al. (2019). That is, we set $\phi^m = 0.25$ in the labor supply specification (26).

In all specifications, we obtain an estimate of ϵ close to four. All panels report high values of ϕ^w and ψ , but point estimates vary across specifications. In Panels C and B, the labor supply elasticity to wages is higher, but productivity elasticity to employment is lower. Finally, notice that our estimate of ϕ^p is much lower in Panels B and C. This is because the estimation of this parameter relies more on the general equilibrium mechanism of the model, whose strength varies with the specification of trade imbalances and labor supply.

We then evaluate how these assumptions affect our estimates of the reduced-form elasticities. This is important because our theoretical results imply that these are the sufficient objects to compute the model’s predicted responses for any given vector of shifts in revenue and consumption costs. For this reason, Table A.9 reports the correlation between the reduced-form elasticities implied by our baseline specification and the alternative specifications shown in Table A.8. We can see that the correlations are very high for all alternative specifications, attesting the robustness of our baseline estimates of the reduced-form elasticities.

Finally, we report the fit coefficients estimated with equation (39) for the alternative specifications in Table A.8. As in our baseline, Table A.10 shows that the fit coefficients are not statistically different from one for all alternative specifications.

Table A.8: Estimates of the Structural Parameters, Robustness

| ψ | ϕ^w | ϕ^p | ϵ |
|--|----------------|-----------------|----------------|
| <i>Panel A: Baseline</i> | | | |
| 0.56 (0.07) | 2.11 (0.25) | -1.36 (0.24) | 3.94 (0.41) |
| <i>Panel B: Allowing for trade imbalances</i> | | | |
| 0.62 (0.07) | 1.92 (0.22) | -0.73 (0.45) | 3.98 (0.40) |
| <i>Panel C: Labor supply normalization with national price index</i> | | | |
| 0.50 (0.07) | 2.92 (0.60) | -0.12 (0.41) | 3.96 (0.35) |
| <i>Panel D: Exogenous labor supply links, $\phi^m = 0.25$</i> | | | |
| 0.35 (0.05) | 3.59 (0.54) | -1.28 (0.99) | 4.42 (0.82) |

Notes: Estimation of θ using reduced-form expressions in (37) for the pooled sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. Estimation uses the two-step procedure in Proposition 5. All specifications include the set of baseline controls in Table 1. Each panel corresponds to one specification of spatial links (as described in the main text). Standard errors in parentheses.

Table A.9: Correlation between Baseline and Alternative Estimates of Reduced-form Elasticities

| | (1) | (2) | (3) |
|---|------|------|------|
| <i>Panel A: Elasticity to revenue shifts</i> | | | |
| β_{ii}^R | 0.93 | 0.94 | 0.96 |
| φ_{ii}^R | 0.92 | 0.94 | 0.95 |
| β_{ij}^R | 0.57 | 0.92 | 0.44 |
| φ_{ij}^R | 0.87 | 0.92 | 0.78 |
| <i>Panel B: Elasticity to consumption cost shifts</i> | | | |
| β_{ii}^C | 0.75 | 0.67 | 0.90 |
| φ_{ii}^C | 0.78 | 0.57 | 0.90 |
| β_{ij}^C | 0.80 | 0.61 | 0.72 |
| φ_{ij}^C | 0.82 | 0.58 | 0.59 |
| <i>Specification</i> | | | |
| Panel B of Table A.8 | x | | |
| Panel C of Table A.8 | | x | |
| Panel D of Table A.8 | | | x |

Notes: Table of correlations between reduced-form elasticities computed with alternative specifications of spatial links and estimated parameters reported in Table A.8. Sample of 521,284 pairs of CZs in 2000.

Table A.10: Predicted Impact of the China Shock and Labor Market Outcomes across U.S. CZs, Robustness

| | (1) | (2) | (3) | (4) |
|--|-------------------|-------------------|-------------------|-------------------|
| <i>Panel A: Change in avg. weekly log wage</i> | | | | |
| Predicted Effect | 0.67** (0.27) | 1.08*** (0.32) | 0.82** (0.34) | 1.07* (0.57) |
| <i>Panel B: Change in log of employment</i> | | | | |
| Predicted Effect | 0.90*** (0.14) | 1.24*** (0.31) | 0.86*** (0.20) | 1.07*** (0.29) |
| <i>Specification</i> | | | | |
| Baseline | x | | | |
| Panel B of Table A.8 | | x | | |
| Panel C of Table A.8 | | | x | |
| Panel D of Table A.8 | | | | x |

Notes: Pooled sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Robust standard errors in parentheses are clustered by state.

A.2.2 Additional Results

Cross-regional variation in reduced-form elasticities. We now investigate the determinants of the cross-section variation in the estimated indirect effects. We first regress $\beta_{ij}^R(\hat{\theta})$ and $\varphi_{ij}^R(\hat{\theta})$ on the bilateral index z_{ij} that we use to construct the indirect effects in the simple extension of ADH in equation (38). Columns (1) and (2) in Table A.11 indicate that regions that are larger or closer to each other have larger reduced form elasticities. That is, they induce stronger indirect effects in general equilibrium. The low R^2 , however, suggest that the gravity bilateral index z_{ij} is not able to capture the vast heterogeneity in the reduced-form elasticities implied by our baseline specification.

Next, we investigate how the different observed components of the spatial links matrix determine the matrices $\beta_{ij}^R(\hat{\theta})$ and $\varphi_{ij}^R(\hat{\theta})$. We build on the series expansion shown in Theorem 2 and look at how the observed bilateral components of $\tilde{\gamma}$ – namely, the revenue share y_{ij} and the revenue elasticity χ_{ij} – predict the estimated reduced-form elasticities. Table A.11 indicates that a large share of the dispersion of indirect effects is explained by these two variable: the R^2 is around 0.5 for both elasticities. For both variables, the indirect effect is increasing in χ_{ij} , indicating that indirect effects are stronger between regions with a similar sectoral and destination composition.²

²Another determinant of the spatial matrix $\tilde{\gamma}$ is the matrix of trade shares \bar{x} . However, since the correlation between y_{ij} and x_{ij} is close to 1, we do not add x_{ij} to the regression.

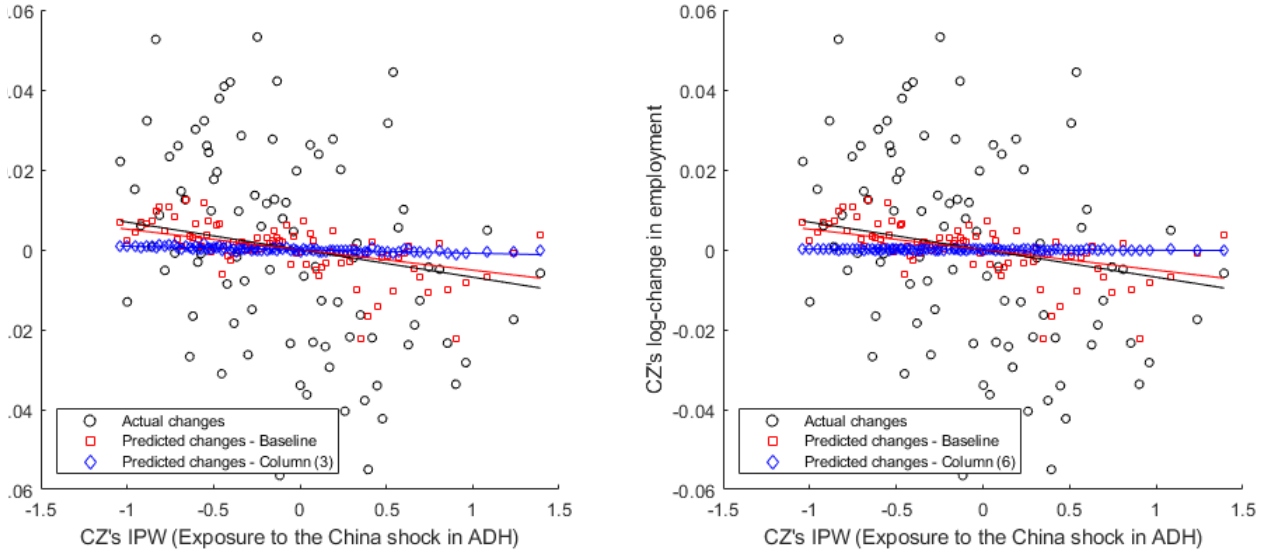
Table A.11: Estimates of Indirect Effects and Observable Spatial Links

| Reduced-Form Elasticity | Wage (β_{ij}^R) (1) | Employment (φ_{ij}^R) (2) | Wage (β_{ij}^R) (3) | Employment (φ_{ij}^R) (4) |
|-------------------------------------|--------------------------------|--|--------------------------------|--|
| Size/Distance (z_{ij}) | 0.027*** (0.00) | 0.055*** (0.00) | | |
| Revenue share (y_{ij}) | | | -2.916*** (0.02) | -1.866*** (0.03) |
| Gravity competition (χ_{ij}) | | | 8.630*** (0.02) | 12.540*** (0.03) |
| Constant | 0.005*** (0.00) | 0.009*** (0.00) | 0.002*** (0.00) | 0.003*** (0.00) |
| R^2 | 0.003 | 0.003 | 0.583 | 0.524 |

Notes: Sample of 468,506 bilateral pairs of Commuting Zones. All the variables are constructed using data for 2000 and the baseline estimates in Panel A of Table 3. We trim the top and bottom 5% of the observations. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Model Fit: Predicted responses and regional shock exposure. Figure A.1 investigates further the ability of the different specifications of spatial links to match the findings in ADH – namely, the magnitude of the differential employment response to higher regional exposure to Chinese import competition. The black circles replicate the relationship in column (1) of Table 1: regions with \$1000 dollars more of per-worker exposure to Chinese import competition (i.e., higher IPW_i^t) suffer an employment growth reduction of 0.56 log-points. The red squares illustrate the relationship between regional shock exposure and employment changes predicted by our baseline estimates. We can see that our baseline model yields a negative differential response to local shock exposure whose magnitude is very similar to that in the data. Notice that the squares are not perfectly aligned with IPW_i^t because employment changes in our model are also driven by regional consumption exposure and indirect effects created by the shock exposure of other CZs. Finally, the blue diamonds indicate that the alternative specifications in columns (3) and (6) of Table 5 are not able to replicate the magnitude of the differential employment response to higher revenue exposure across CZs. They predict that regions in which IPW_i^t is \$1000 dollars higher experience reductions in employment growth of -0.089 and -0.017 log-points according to the specifications in columns (3) and (6), respectively.

Figure A.1: Employment Changes and Exposure to Chinese Import Competition across U.S. CZs



Notes: Bin scatter plot of changes in log of employment and exposure to Chinese import competition (IPW_i^t) after partialling out the baseline controls in Table 1. Pooled sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. Plots report average log-employment change and IPW_i^t by percentile bins based on IPW_i^t . Baseline predicted changes are computed with the reduced-form responses in equation (37) using estimates in Panel A of Table 3. Alternative calibration computed with the reduced-form responses in equation (37) using parameters in the corresponding columns of Table 5.

A.3 The Impact of the China Shock in General Equilibrium

A.3.1 Robustness

Table A.12 computes the effect of the China shock using the alternative specifications of the spatial links discussed in Section A.2.2. The first row of each panel reports the predicted responses implied by our baseline specification. For each alternative specification, the remaining rows present the average, standard deviation and correlation with baseline predicted responses across CZs.

Our results indicate that all alternative specifications yield predicted responses that have a high correlation with our baseline predicted responses. The correlations reported in the third column are always between 0.79 and 0.89. However, the first column indicates that alternative specifications generate average responses that may be higher or lower than our baseline predicted responses. The average negative effect of the shock is stronger when we allow for trade imbalances or specify stronger labor supply links. This is because of the different estimated parameters reported in Table 3 that are necessary to match the direct and indirect impacts of shock exposure on labor market outcomes. We obtain a higher agglomeration elasticity in the case of trade imbalances and a higher labor supply elasticity in the case of exogenous migration links. These tend to strengthen the amplification channel created by the combination of endogenous responses in employment and productivity. Lastly, the second row of Panel B shows that the average employment response is smaller when we specify the labor supply normalization in terms of the U.S. price index. This is because the national employment change is now more sensitive to the decline in import prices.

Table A.12: Effect of the China Shock on U.S. CZs, 1990-2007 – Robustness

| | National Average | Standard Deviation | Correlation w/ baseline |
|----------------------------|---------------------|-----------------------|----------------------------|
| <i>Panel A: Wages</i> | | | |
| Baseline | -3.98 | 1.30 | 1.00 |
| Panel B of Table A.8 | -5.64 | 1.34 | 0.83 |
| Panel C of Table A.8 | -3.37 | 1.12 | 0.89 |
| Panel D of Table A.8 | -3.69 | 0.65 | 0.88 |
| <i>Panel B: Employment</i> | | | |
| Baseline | -2.78 | 3.31 | 1.00 |
| Panel B of Table A.8 | -4.08 | 2.71 | 0.79 |
| Panel C of Table A.8 | -0.52 | 3.30 | 0.84 |
| Panel D of Table A.8 | -5.44 | 2.39 | 0.84 |
| <i>Panel C: Real Wages</i> | | | |
| Baseline | 0.16 | 1.75 | 1.00 |
| Panel B of Table A.8 | -0.79 | 1.57 | 0.79 |
| Panel C of Table A.8 | 1.30 | 1.40 | 0.86 |
| Panel D of Table A.8 | -0.12 | 0.81 | 0.85 |

Notes: Predicted changes in employment and wages computed with the reduced-form responses in equation (37). Predicted real wage change computed with expression (23). Reduced-form elasticities computed with alternative specifications of spatial links and estimated parameters reported in Table A.8.

A.3.2 Additional Results

We first report the national effect of the China shock on wages, employment and real wages, separately for the two time periods. Tables A.13 and A.14 document that in the first decade of our analysis, between 1990 and 2000, the effect of the rise of China has been much smaller than the effect in the second decade. This is a consequence of the larger magnitude of the shock in the second period since China increased its exports to high-income countries more after its accession to WTO in 2001. The tables also show that the indirect effects reinforce the direct effects of both revenue and consumption shifts in both periods. Lastly, we can see that the predicted responses of employment were more dispersed than the ones of wages throughout both decades.

Table A.13: Effect of the China Shock on U.S. CZs, 1990-2000

| | Wage | | Employment | | Real wage | |
|-----------------------------|---------|----------|------------|----------|-----------|----------|
| | Average | St. Dev. | Average | St. Dev. | Average | St. Dev. |
| Total effect | -1.14 | 0.70 | 0.06 | 1.91 | 0.51 | 1.02 |
| Direct effect of η^R | -0.31 | 1.03 | -0.76 | 2.82 | -0.39 | 1.51 |
| Direct effect of η^C | 0.32 | 0.65 | 1.03 | 1.86 | 0.99 | 1.03 |
| Indirect effect of η^R | -0.92 | 0.95 | 0.28 | 2.70 | 0.03 | 1.47 |
| Indirect effect of η^C | -0.23 | 0.58 | -0.49 | 1.66 | -0.13 | 0.90 |

Notes: Predicted changes in employment and wages computed with the reduced-form responses in equation (37) using estimates in Panel A of Table 3. Predicted real wage change computed with expression (23).

Table A.14: Effect of the China Shock on U.S. CZs, 2000-2007

| | Wage | | Employment | | Real wage | |
|-----------------------------|---------|----------|------------|----------|-----------|----------|
| | Average | St. Dev. | Average | St. Dev. | Average | St. Dev. |
| Total effect | -2.84 | 0.82 | -2.85 | 2.03 | -0.33 | 1.06 |
| Direct effect of η^R | -0.50 | 0.84 | -1.18 | 2.15 | -0.59 | 1.13 |
| Direct effect of η^C | 0.66 | 0.77 | 2.15 | 2.23 | 2.15 | 1.30 |
| Indirect effect of η^R | -3.32 | 1.05 | -5.23 | 2.72 | -2.91 | 1.43 |
| Indirect effect of η^C | 0.32 | 0.68 | 1.41 | 1.94 | 1.01 | 1.05 |

Notes: Predicted changes in employment and wages computed with the reduced-form responses in equation (37) using estimates in Panel A of Table 3. Predicted real wage change computed with expression (23).

B Online Appendix: Micro-foundations and Welfare with Endogenous Labor Supply

In this appendix, we establish micro-foundations for the functions governing the spatial links in labor supply and labor productivity in our model. We then use one of these micro-foundations to derive the equivalent welfare variation implied by a shock as a function of the shock's impact on the real wage in each market.

B.1 Labor Supply Function

In this section, we lay out three settings that imply labor supply functions of the form in equation (1). In the first framework, we derive the labor supply function from a setting in which a representative household decides the number of hours worked (intensive margin) in each region of the country. The second framework yields the labor supply function from the choice of heterogeneous individuals of working and residing in different regions of the country (extensive margin). In the third framework, we derive a special case of equation (1) in a Roy model with a generic distribution of individual-specific amenities and efficiencies in different sector-region pairs.

B.1.1 Representative Household with Intensive Labor Supply Margin

We lay out a setting in which a representative household decides the number of hours worked in each region of the country. This is the same mechanism present in business cycle models with elastic labor supply, see e.g. Shimer (2009) and Keane (2011). We consider an extended version of this framework where the representative household allocates individuals across different regions within the country.

Preferences. Country c has a representative household with preferences over consumption and labor supply in different markets. The representative agent's utility function is given by

$$U(\{N_i V_i(C_i, H_i)\}_i), \quad (\text{B.1})$$

where N_i is the total number of individuals in region i and V_i is the utility of individuals in region i . We assume that $U_c(\cdot)$ is twice differentiable, and strictly quasi-concave.

We assume that V_i takes the following separable form:

$$V_i(C_i, H_i) = \frac{(C_i)^{1-\eta}}{1-\eta} - \frac{H_i^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} \quad (\text{B.2})$$

where H_i is the per-worker hours worked in region i and C_i is a consumption aggregator. This utility function has two parameters: $\eta > 0$ regulates income effects, while $\phi > 0$ is the Frisch elasticity of labor supply. This specification has been used by Mankiw et al. (1985), Domeij and Floden (2006), Corsetti et al. (2007) and Keane (2011) among others. In the extreme case of $\eta = 0$, income effects are absent and thus labor is increasing in real wage, while if $\eta > 1$ the income effect dominates the substitution effect and labor supply is decreasing in the real wage. The limiting case of $\eta = 1$ is often used in the macro literature, such as Kimball and Shapiro (2008), Shimer (2009) and Ohanian and Raffo (2012), because in this way preferences are consistent with a balanced growth path. We further assume that C_i takes the nested CES form that yields the spending shares in (6) and the price index in (8).

We impose that country c has \bar{N}_c individuals such that

$$\sum_{i \in c} N_i = \bar{N}_c.$$

Budget Constraint. In each region, consumption is financed by labor income and a lump-sum transfer of \tilde{T}_i units, expressed in terms of a numeraire $b(\mathbf{w}, \mathbf{P})$. There is also an income tax of $\tilde{t}\%$ of total income (including the transfer). Thus, the budget constraint in region i is

$$N_i C_i P_i = (1 - \tilde{t}) \left(w_i H_i N_i + \tilde{T}_i b(\mathbf{w}, \mathbf{P}) N_i \right).$$

where we impose that $b(\mathbf{w}, \mathbf{P})$ is homogeneous of degree one in (\mathbf{w}, \mathbf{P}) .

We impose budget balance at the country-level, so that

$$\tilde{t} \sum_{i \in c} w_i H_i N_i = (1 - \tilde{t}) \sum_{i \in c} \tilde{T}_i b(\mathbf{w}, \mathbf{P}) N_i$$

This can be rearranged to obtain the optimal tax rate

$$\tilde{t} = \frac{b(\mathbf{w}, \mathbf{P}) \sum_{i \in c} \tilde{T}_i N_i}{b(\mathbf{w}, \mathbf{P}) \sum_{i \in c} \tilde{T}_i N_i + \sum_{i \in c} w_i H_i N_i}$$

Intensive margin of labor supply in each region. We first solve the first-stage problem of deciding the number of hours worked in each region i conditional on N_i :

$$V_i \left((1 - \tilde{t}) \frac{w_i}{P_i}, \tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i} \right) \equiv \max_{H_i} \frac{1}{1 - \eta} (1 - \tilde{t}) \left(\frac{w_i}{P_i} H_i + \tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i} \right)^{1 - \eta} - \frac{H_i^{1 + \frac{1}{\phi}}}{1 + \frac{1}{\phi}}. \quad (\text{B.3})$$

The first order condition of this problem implies that

$$\left((1 - \tilde{t}) \frac{w_i}{P_i} \right)^{\frac{1}{\eta}} H_i^{-\frac{1}{\eta\phi}} - (1 - \tilde{t}) \frac{w_i}{P_i} H_i = (1 - \tilde{t}) \tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i}$$

which implies that

$$H_i = \Phi^H \left((1 - \tilde{t}) \frac{w_i}{P_i}, (1 - \tilde{t}) \tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i} \right) \quad (\text{B.4})$$

It is easy to verify that, if $\eta < 1$, then $\frac{\partial \Phi^H}{\partial (1 - \tilde{t}) \frac{w_i}{P_i}} > 0$ and $\frac{\partial \Phi^H}{\partial (1 - \tilde{t}) \tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i}} < 0$. We obtain the indirect utility in region i by substituting H_i in (B.4) into the objective function in (B.3).

Number of individuals in each region. Finally, the second-stage problem is

$$\max_{N_i} U_c \left(\left\{ V_i \left((1 - \tilde{t}) \frac{w_i}{P_i}, (1 - \tilde{t}) \tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i} \right) N_i \right\}_i \right) \quad \text{s.t.} \quad \sum_i N_i = \bar{N}_c$$

We can equate the first-order condition for any two regions:

$$\frac{\partial U}{\partial N_1} V_1 \left((1 - \tilde{t}) \frac{w_1}{P_1}, (1 - \tilde{t}) \tilde{T}_1 \frac{b(\mathbf{w}, \mathbf{P})}{P_1} \right) = \frac{\partial U}{\partial N_i} V_i \left((1 - \tilde{t}) \frac{w_i}{P_i}, (1 - \tilde{t}) \tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i} \right) \quad \forall i \in c \quad (\text{B.5})$$

where the tax rate \tilde{t}_i is given by

$$\tilde{t} = \frac{\sum_{i \in c} \frac{b(\mathbf{w}, \mathbf{P})}{P_i} \tilde{T}_i N_i}{\sum_{i \in c} \frac{b(\mathbf{w}, \mathbf{P})}{P_i} \tilde{T}_i N_i + \sum_{i \in c} \frac{w_i}{P_i} H_i N_i}, \quad (\text{B.6})$$

and the total population constraint is given by

$$\sum_{i \in c} N_i = \bar{N}_c. \quad (\text{B.7})$$

In equilibrium, N_i and \tilde{t} solve the system in (B.5)–(B.7). Notice that the system implies N_i and \tilde{t} are functions of w_i/P_i and $\tilde{T}_i b(\mathbf{w}, \mathbf{P})/P_i$. Thus, we can write

$$N_i = \Phi^N \left(\left\{ \frac{w_i}{P_i}, \tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i} \right\}_i \right) \quad \text{and} \quad \tilde{t} = \Phi^{tax} \left(\left\{ \frac{w_i}{P_i}, \tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i} \right\}_i \right). \quad (\text{B.8})$$

Labor supply. Labor supply in region i is $L_i = H_i N_i$. The combination of equations (B.4) and (B.8) implies that

$$L_i = \Phi_i^L \left(\left\{ \frac{w_i}{P_i}, \tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i} \right\}_i \right) \quad (\text{B.9})$$

and, therefore, it has the general form in equation (1).

Notice that, as long $\eta < 1$, both H_i and N_i respond to $\tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i}$, so that $\phi_{ij}^w \neq -\phi_{ij}^p$. This becomes clear in the case of exogenous population in each region where labor supply is given by (B.4). In fact, in this case, we have that

$$\frac{\partial \ln H_i}{\partial \ln P_i} = -\frac{\partial \ln H_i}{\partial \ln w_i} + \frac{\partial \ln \Phi^H}{\partial \ln T_i} \tilde{T}_i \frac{b(\mathbf{w}, \mathbf{P})}{P_i H_i} \left(1 + \frac{\partial \ln b}{\partial \ln P_i} \right),$$

so that $-\frac{\partial \ln H_i}{\partial \ln P_i} < \frac{\partial \ln H_i}{\partial \ln w_i}$ whenever $\frac{\partial \ln b}{\partial \ln P_i} \geq 0$.

B.1.2 Spatial Assignment with Extensive Labor Supply Margin

We now present a micro-foundation for the general specification of the labor supply equation in Section 2 based on the choice of heterogeneous agents to work and reside in different regions of a country. Our setting combines the extensive margin decision to work of heterogeneous individuals in terms of disutility to work, as in Rogerson (1988) and Chetty (2012), and the discrete choice of residing in different regions, as in Bryan and Morten (2015). A similar framework has been used in Adão, Kolesár and Morales (2019) to motivate shift-share empirical designs.

Preferences. Consider a country c composed of regions (indexed by i and j). We assume that all individuals in the country have identical preferences for consumption goods, which take the nested CES structure that generates the trade shares introduced in Section 2. Each region i is endowed with \bar{N}_i individuals that have heterogeneous preferences for the amenities of the different regions of the country, $\mathbf{a} \equiv \{a_j\}_j$. If an individual moves from region i to region j , her amenity value is discounted by ζ_{ij} . Individuals residing in region j may decide to work at a wage rate w_j or to stay at home in exchange for a government transfer of b_j . All residents of region j receive a lump-sum transfer $t_j = \chi b_j$ and pay an ad-valorem income tax χ . Then, once an individual resides in region i , she learns her work disutility ν and decides whether or not to participate in the labor market. Thus, the utility of an individual born in

region i living in region j is

$$U_{ij}(a, \nu) = \begin{cases} \zeta_{ij} a_j \frac{\nu w_j \chi + t_j}{P_j} & \text{if work} \\ \zeta_{ij} a_j \frac{b_i \chi + t_i}{P_i} & \text{if do not work} \end{cases}$$

where P_i is the price index in region j .

We assume that individuals take i.i.d. draws of their amenity vector and their work disutility such that

$$a_j \sim e^{-a^{-\phi^m}} \quad \text{and} \quad \nu \sim 1 - (\phi^e - 1)v^{-\phi^e},$$

where $\phi^m > 0$, $\phi^e > 1$, and $\nu \geq (\phi^e - 1)^{1/\phi^e}$.

Budget Constraint. In equilibrium, we assume that the income tax rate χ guarantees budget balance in the country. Let N_j be the population of region j and e_j the employment share in region j . The budget balance condition requires that

$$\chi = \frac{\sum_i b_i (1 - e_i) N_i}{\sum_i (w_i - b_i) e_i N_i}.$$

Extensive margin of labor supply in each region. We now derive the labor supply equation in each region j . Among the N_j residents of region j , only individuals with $\nu w_j > b_j$ decide to work. Thus, the employment share in region j is

$$e_j = Pr[\nu w_j > b_j] = (\phi^e - 1) \left(\frac{w_j}{b_j} \right)^{\phi^e}. \quad (\text{B.10})$$

Number of individuals in each region. This implies that the expected utility of an individual born in region i and residing in region j is $\bar{U}_{ij}(a) \equiv E[U_{ij}(a, \nu)]$, which is equivalent to

$$\begin{aligned} \bar{U}_{ij}(a) &= \frac{\zeta_{ij} a_j}{P_j} \left(\chi w_j (\phi^e - 1) \int_{b_j/w_j}^{\infty} \phi^e v^{-\phi^e} dv + \chi b_j (\phi^e - 1) \int_{(\phi^e - 1)^{1/\phi^e}}^{b_j/w_j} \phi^e v^{-\phi^e - 1} dv + t_j \right) \\ &= \frac{\zeta_{ij} a_j}{P_j} \left(-\chi w_j \phi^e \left(\lim_{v \rightarrow \infty} (v)^{1-\phi^e} - \left(\frac{b_j}{w_j} \right)^{1-\phi^e} \right) - \chi b_j (\phi^e - 1) \left(\left(\frac{b_j}{w_j} \right)^{1-\phi^e} - (\phi^e - 1)^{-1} \right) + t_j \right) \\ &= \frac{\zeta_{ij} a_j}{P_j} \left((\chi \phi^e - \chi (\phi^e - 1)) w_j^{\phi^e} b_j^{1-\phi^e} - \chi b_j + t_j \right) \\ &= \left(\chi \frac{\zeta_{ij} w_j^{\phi^e} b_j^{1-\phi^e}}{P_j} \right) a_j \end{aligned}$$

where the last row uses the fact that we specify the lump-sum transfer to be $t_j = \chi b_i$. This implies that the share of individuals born in region i residing in region j is

$$n_{ij} = Pr \left[j = \arg \max_{j'} \left\{ \left(\chi \frac{\zeta_{ij'} w_{j'}^{\phi^e} b_{j'}^{1-\phi^e}}{P_{j'}} \right) a_{j'} \right\} \right] = \frac{\left(\frac{\zeta_{ij} w_j^{\phi^e} b_j^{1-\phi^e}}{P_j} \right)^{\phi^m}}{\sum_{j'} \left(\frac{\zeta_{ij'} w_{j'}^{\phi^e} b_{j'}^{1-\phi^e}}{P_{j'}} \right)^{\phi^m}}. \quad (\text{B.11})$$

Thus, the number of individuals residing in region j is $N_j = \sum_i n_{ij} \bar{N}_i$

Labor supply. Thus, given $\{w_j, b_j, P_j\}_j$, total employment in region j is $L_j = e_i \sum_j n_{ij} \bar{N}_i$, which is equivalent to

$$L_j = (\phi^e - 1) \left(\frac{w_j}{b_j} \right)^{\phi^e} \sum_i \frac{\left(\frac{\zeta_{ij} w_j^{\phi^e} b_j^{1-\phi^e}}{P_j} \right)^{\phi^m}}{\sum_{j'} \left(\frac{\zeta_{ij'} w_{j'}^{\phi^e} b_{j'}^{1-\phi^e}}{P_{j'}} \right)^{\phi^m}} \bar{N}_i.$$

To derive the labor supply equation in terms of wages and prices, it is necessary to specify the monetary non-employment benefit b_i . We assume that the benefit in region i has a component that depends on local outcomes and another that is common to all regions:

$$b_j = \tilde{b}_j w_j^{\alpha_w} P_j^{\alpha_p} b(\mathbf{w}, \mathbf{P})^{1-\alpha_w-\alpha_p},$$

where $b(\mathbf{w}, \mathbf{P})$ is homogeneous of degree one in (\mathbf{w}, \mathbf{P}) . In this specification, \tilde{b}_i is an exogenous shifter of the benefit and, therefore, captures the generosity of transfers available in each region. The other components specify how the transfer responds to the outcomes in the own region and those in the overall economy. Notice that, by imposing restrictions that guarantee that the benefit is homogeneous of degree one in (\mathbf{w}, \mathbf{P}) , we prevent the specification of the economy's numeraire from affecting employment. Under this assumption, labor supply in region i is

$$L_j = v_j \left(\frac{w_j^{\phi^{e,w}} P_j^{\phi^{e,p}}}{b(\mathbf{w}, \mathbf{P})^{\phi^{e,w} + \phi^{e,p}}} \right) \sum_i \frac{\zeta_{ij} w_j^{\tilde{\phi}^w} P_j^{\tilde{\phi}^p}}{\sum_{j'} \zeta_{ij'} w_{j'}^{\tilde{\phi}^w} P_{j'}^{\tilde{\phi}^p}} \bar{L}_i, \quad (\text{B.12})$$

where $v_j \equiv (\phi^e - 1) \tilde{b}_j^{-\phi^e}$, $\phi^{e,w} \equiv (1 - \alpha_w) \phi^e$, $\phi^{e,p} \equiv -\alpha_p \phi^e$, $\tilde{\phi}^w \equiv (\phi^e + (1 - \phi^e) \alpha_w) \phi^m$, $\tilde{\phi}^p \equiv -(1 + \alpha_p (\phi^e - 1)) \phi^m$.

This expression implies that labor supply takes the general form specified in equation (1) where $\phi_{ij}^w \neq -\phi_{ij}^p$ whenever $\alpha_p < 1$. In the case of $\alpha_p = 0$ and $\phi^m = 0$,

$$L_j = v_j \left(\frac{w_j}{b(\mathbf{w}, \mathbf{P})} \right)^{\phi^e (1 - \alpha_w)},$$

so that $\frac{\partial \ln L_i}{\partial \ln P_i} = -\phi^e (1 - \alpha_w) \frac{\partial \ln b}{\partial \ln P_i}$ and $\frac{\partial \ln L_i}{\partial \ln w_i} = \phi^e (1 - \alpha_w) \left(1 - \frac{\partial \ln b}{\partial \ln w_i} \right)$. Thus, $0 \leq -\frac{\partial \ln L_i}{\partial \ln P_i} < \frac{\partial \ln L_i}{\partial \ln w_i}$ whenever $0 \leq \frac{\partial \ln b}{\partial \ln P_i} < 1 - \frac{\partial \ln b}{\partial \ln w_i}$.

B.1.3 Spatial Assignment Models with General Distribution of Amenities and Efficiency

In this section, we consider a general class of assignment models in which a market is defined as a group of industries in a region. Individuals are heterogeneous in terms of both market-specific productivity and preferences. Our setting covers the seminal setting in Roy (1951) and its recent applications in international trade – e.g., Adão (2015), Burstein et al. (2019), Galle et al. (2017), and Lee (2020).

Preferences. Suppose that countries are populated by a continuum of individuals, $\iota \in I_c$, that are heterogeneous in terms of preferences and efficiency across markets (i.e, sector-region pairs). We assume individual ι has market specific preferences, $a_j(\iota)$, and market specific efficiency, $e_j(\iota)$. In particular, if employed in market j , we assume that individual ι has homothetic preferences given by

$$U_j(\iota) = a_j(\iota) + C_j,$$

where C_j is a consumption aggregator that is identical to all individuals. We assume that it takes the nested CES structure that generates the trade shares introduced in Section 2. Notice that, compared to the models presented in the previous two sections, the agents cannot choose to work for the home sector. Thus the model is simpler in that we do not need to specify the unemployment benefit function and the tax system.

We further assume that individuals take independent draws of $(a_j(\iota), e_j(\iota))$ from a common full support distribution:

$$\{a_j(\iota), e_j(\iota)\}_j \sim F(\mathbf{a}, \mathbf{e}).$$

Notice that, compared to the framework in the previous section, we allow here for a general distribution $F(\mathbf{a}, \mathbf{e})$ that governs cross-section heterogeneity in both market-specific amenities and efficiency across individuals. It covers as special case of extreme value distributions used in Burstein et al. (2019), Galle et al. (2017), and Lee (2020). It also covers the setting in Adão (2015) that only entails heterogeneity in sector-specific efficiency.

Budget Constraint. The budget constraint of individual ι is given by

$$C_j = \frac{w_j e_j(\iota)}{P_j}$$

where P_j is the price index in (8). To simplify the notation, we let $\omega_j = w_j/P_j$ be the real wage in market j .

Labor supply. The solution of this problem implies that, for individual ι , the utility of being employed in j is $U_j(\iota) = a_j(\iota) + \omega_j e_j(\iota)$. Thus, the set of individuals choosing j is

$$I_j(\{\omega_i\}_i) \equiv \{(\mathbf{a}, \mathbf{e}) : a_j + e_j \omega_j \geq a_i + e_i \omega_i \forall i\}.$$

Thus, the labor supply is a function of the real wage vector in the economy:

$$L_j = \Phi_j(\boldsymbol{\omega}) \equiv \int_{I_j(\{\omega_i\}_i)} e_j dF_c(\mathbf{a}, \mathbf{e}). \quad (\text{B.13})$$

Notice that the function $\Phi_j(\cdot)$ is homogeneous of degree zero with $\frac{\partial \Phi_j}{\partial \omega_j} > 0$ and $\frac{\partial \Phi_j}{\partial \omega_i} < 0$.³ As shown in Adão (2015), these properties imply that $\Phi(\cdot)$ is invertible up to scalar. This expression is a special case of the labor supply function in (1) where $\phi_{ij}^w = -\phi_{ij}^p$ for all i and j .

B.2 Labor Productivity

In this section, we provide a micro-foundation for the labor productivity function in (5) from a spatial model with capital and land in production.

B.2.1 Model with Other Factors in Production

Preferences. Consider a country c composed of regions and sectors. We assume that all individuals in the country have identical preferences for consumption goods, which take the nested CES structure

³The homogeneity of $\Phi_j(\cdot)$ follows immediately from the definition of I_j . To see that $\frac{\partial \Phi_j}{\partial \omega_j} \geq 0$ and $\frac{\partial \Phi_j}{\partial \omega_i} \leq 0$, notice that $I_i(\tilde{\omega}_c) \subset I_i(\omega_c)$ and $I_j(\omega_c) \subset I_j(\tilde{\omega}_c)$ whenever $\tilde{\omega}_j > \omega_j$ and $\tilde{\omega}_i = \omega_i$.

that generates the trade shares introduced in Section 2. We assume that labor supply is generated by the class of spatial assignment modes in Section B.1.3. In this case, labor supply is a function of the vector of after-tax real wages in the country:

$$L_i = \Phi_i \left(\left\{ (1 + \rho_j) \frac{w_j}{P_j} \right\}_j \right),$$

where ρ_i is an ad-valorem income tax rate in region i .

Production. Each sector has a production function that is linear in a regional composite good:

$$Q_{i,s} = \zeta_{i,s} Q_i.$$

The production function of the composite good in each region uses labor, capital, and land:

$$Q_i = \Psi_i^Q(\mathbf{L}) \left(\frac{K_i}{\alpha_i^K} \right)^{\alpha_i^K} \left(\frac{L_i}{\alpha_i^L} \right)^{\alpha_i^L} \left(\frac{T_i}{\alpha_i^T} \right)^{\alpha_i^T} \quad (\text{B.14})$$

where $\alpha_i^L + \alpha_i^K + \alpha_i^T = 1$.

Endowments of capital and land. Land is immobile across regions. Each region has an endowment \bar{T}_i of land. In contrast, we assume that capital is fully mobile across regions. The country has a capital endowment of \bar{K}_c .

Similar to Caliendo et al. (2018), there is a national mutual fund that owns all the land and capital endowments of the country. We assume that the central government distributes dividends to workers in different regions such that

$$1 + \rho_i = P_i \chi.$$

This implies that the labor supply in region i is

$$L_i = \Phi_i(\{\chi w_i\}_i). \quad (\text{B.15})$$

The tax rate χ guarantees budget balance:

$$\sum_i (\chi P_i - 1) w_i L_i = \sum_i R_i \bar{T}_i + \sum_i r_i \bar{K}_i. \quad (\text{B.16})$$

Labor productivity function. Profit maximization implies that the price of the composite intermediate good in region i is

$$p_i = \frac{1}{\Psi_i^Q(\mathbf{L})} (w_i)^{\alpha_i^L} (R_i)^{\alpha_i^T} (r_i)^{\alpha_i^K} \quad (\text{B.17})$$

To obtain the equilibrium level of R_i , consider the market clearing condition for the land in region i . Using the fact that the regional spending shares on land and labor are α_i^T and α_i^L , we have that

$$\bar{T}_i = T_i = \frac{\alpha_i^T}{\alpha_i^L} \frac{w_i L_i}{R_i}$$

and, therefore,

$$R_i = \frac{\alpha_i^T}{\alpha_i^L} \frac{w_i L_i}{\bar{T}_i} \quad (\text{B.18})$$

Since capital is fully mobile across regions, its rental price is identical everywhere: $r_i = r_c$ for all i . The rental rate must satisfy the capital market clearing condition, $r_c \bar{K} = \sum_i r_c K_i$. Using the fact that a share α_i^K of the regional revenue is spent on capital, we get the following expression for the equilibrium rent:

$$r_c = \frac{1}{\bar{K}_c} \sum_i \frac{\alpha_i^K}{\alpha_i^L} w_i L_i. \quad (\text{B.19})$$

By plugging the expressions for R_i and r_c in (B.18)–(B.19) into the expression for p_i in (B.17), we get that

$$p_i = \frac{w_i}{\Psi_i^Q(\mathbf{L})} \left(\frac{\alpha_i^T L_i}{\alpha_i^L \bar{T}_i} \right)^{\alpha_i^T} \left(\frac{1}{\bar{K}} \sum_j \frac{\alpha_j^K}{\alpha_j^L} \frac{w_j}{w_i} L_j \right)^{\alpha_i^K}$$

As argued in Section B.1.3, the labor supply equation in (B.15) is invertible up to a scalar, so we can write $\frac{\chi w_j}{\chi w_i} = \Phi_{i,j}^{-1}(\mathbf{L})$. Thus,

$$p_i = \frac{w_i}{\Psi_i^Q(\mathbf{L})} \left(\frac{\alpha_i^T L_i}{\alpha_i^L \bar{T}_i} \right)^{\alpha_i^T} \left(\frac{1}{\bar{K}} \sum_j \frac{\alpha_j^K}{\alpha_j^L} \Phi_{i,j}^{-1}(\mathbf{L}) L_j \right)^{\alpha_i^K},$$

which implies that

$$p_i = \frac{w_i}{\Psi_i(\mathbf{L})}$$

where

$$\tilde{\Psi}_i(\mathbf{L}, \mathbf{P}) \equiv \Psi_i^Q(\mathbf{L}) \left(\frac{\alpha_i^T L_i}{\alpha_i^L \bar{T}_i} \right)^{-\alpha_i^T} \left(\frac{1}{\bar{K}} \sum_j \frac{\alpha_j^K}{\alpha_j^L} \Phi_{i,j}^{-1}(\mathbf{L}) L_j \right)^{-\alpha_i^K}.$$

For completeness, we can also derive the tax rate χ from the budget balance condition in (B.16). Using the fact that shares α_i^T and α^K are spent on land and capital, we can write

$$\chi = \frac{\sum_i \frac{w_i L_i}{\alpha_i^L}}{\sum_i P_i w_i L_i}.$$

B.3 Welfare Gains with Endogenous Labor Supply

B.3.1 Equivalent variation and Real Wages

We consider an environment similar to that of Section B.1.1, but assume that each market has an exogenous number of individuals. Specifically, we consider a representative household in each market i whose homothetic preferences over consumption and hours worked are given by the utility function $V_i(C, H)$. We assume that $V_i(C, H)$ is differentiable, strictly quasi-concave, and strictly increasing in $(C, -H)$. We further assume that C is a consumption aggregator that takes the nested CES form leading to the spending shares in (6) and the price index in (8).

In general, given a price index P_i , a nominal wage w_i and an exogenous transfer T_i , the budget constraint of the representative household in market i is $P_i C_i = w_i H_i + T_i$. Thus, the representative household in market i solves the following utility maximization problem:

$$V_i^* \left(\frac{w_i}{P_i}, \frac{T_i}{P_i} \right) \equiv \max_H V_i \left(\frac{w_i}{P_i} H + \frac{T_i}{P_i}, H \right). \quad (\text{B.20})$$

We consider the welfare consequences of an arbitrary change in prices and wages from (w_i^0, P_i^0) to (w_i, P_i) in an economy without transfers (as in our baseline model, $T_i^0 = T_i = 0$). Following Mas-Colell et al. (1995), we define the Equivalent Variation (EV) as the transfer that equates the welfare in the two equilibria:

$$V_i^* \left(\frac{w_i^0}{P_i^0}, \frac{EV}{P_i^0} \right) = V_i^* \left(\frac{w_i}{P_i}, 0 \right). \quad (\text{B.21})$$

We now show that the equivalent variation is increasing in the real wage change. Let us define the real wage change as $\hat{w}_i = (w_i/P_i)/(w_i^0/P_i^0)$ and the following function whose root defines the equivalent variation,

$$F \left(\frac{EV}{P_i^0}, \hat{w}_i \right) = V_i^* \left(\frac{w_i^0}{P_i^0}, \frac{EV}{P_i^0} \right) - V_i^* \left(\frac{w_i^0}{P_i^0} \hat{w}_i, 0 \right).$$

By the Implicit Function Theorem, there exists a local function $EV(\hat{w}_i)$ such that

$$\frac{\partial EV}{\partial \hat{w}_i} = - \frac{\frac{\partial F}{\partial \hat{w}_i}}{\frac{\partial F}{\partial EV}} = \frac{\frac{\partial V_i^* \left(\frac{w_i^0}{P_i^0} \hat{w}_i, \frac{T_i}{P_i} \right)}{\partial (w/P)} \frac{w_i^0}{P_i^0}}{\frac{\partial V_i^* \left(\frac{w_i^0}{P_i^0}, \frac{T_i+EV}{P_i^0} \right)}{\partial (T/P)} \frac{1}{P_i^0}} = \frac{\frac{\partial V_i(C_i, H_i)}{\partial C}}{\frac{\partial V_i(C_i^{EV}, H_i^{EV})}{\partial C}} H_i w_i^0,$$

where the last equality follows from the Envelope Theorem applied to (B.20). Since $V_i(C, H)$ is increasing in C , this establishes that the equivalent welfare variation is increasing in the real wage change for an arbitrary change in wages and prices.

B.3.2 Special Case with Quasi-Linear Preferences

We now consider a special case in which we derive in closed form the equivalent variation as a share of initial income for an arbitrary change in wages and prices. We consider the same environment as above, but preferences are quasi-linear in consumption:

$$V_i(C_i, H_i) = C_i - \frac{H_i^{1+\frac{1}{\phi}}}{1 + \frac{1}{\phi}}.$$

In this case, the utility maximization problem implies that

$$H_i = \left(\frac{w_i}{P_i} \right)^\phi \quad \text{and} \quad C_i = \left(\frac{w_i}{P_i} \right)^{1+\phi} + \frac{T_i}{P_i},$$

so that the indirect utility in market i is

$$V_i^* \left(\frac{w_i}{P_i}, \frac{T_i}{P_i} \right) = \frac{1}{1+\phi} \left(\frac{w_i}{P_i} \right)^{1+\phi} + \frac{T_i}{P_i}.$$

Thus, after some manipulation, the definition of the equivalent variation in (B.21) implies that

$$\frac{EV}{w_i^0 H_i^0} = \frac{1}{1+\phi} \left[(\hat{w}_i)^{1+\phi} - 1 \right], \quad (\text{B.22})$$

which is increasing in the real wage change.

C Online Appendix: Extensions

We now derive the excess demand shift and the spatial links matrix under extended versions of the baseline model. We first allow for trade imbalances in our baseline model, as in Dekle et al. (2007). Second, we allow for bilateral migration, as in Bryan and Morten (2015). Third, we allow for multiple labor types in each region. Fourth, we extend our demand structure to cover a more general class of one-factor neoclassical models, as in Adao et al. (2017), but augmented to have spatial links in labor supply and labor productivity. Lastly, we consider a model with input-output linkages in production, as Caliendo and Parro (2015), but again extended to have spatial links in labor supply and labor productivity.

C.1 Model with Trade Imbalances

C.1.1 Environment

We extend the baseline model in Section 2 by allowing for exogenous trade imbalances, as in Dekle et al. (2007). Specifically, the trade balance condition is

$$E_j = w_j L_j + T_j, \quad \forall j, \quad (\text{C.1})$$

where the sum of all transfers in the world economy is equal to zero, $\sum_j T_j = 0$. We assume that transfers are set in terms of the world's average wage. As pointed out by Dekle et al. (2007), this implies that transfer changes are invariant to the numeraire choice. Specifically, we impose that $T_j = T(\mathbf{w})\tilde{T}_j$ with \tilde{T}_j and $T(\mathbf{w})$ denoting respectively the number of transfer claims and the world's output.

In this case, the market clearing condition is

$$w_i L_i = \sum_j x_{ij} (w_j L_j + T_j).$$

Equilibrium. To define the equilibrium wage vector, we combine the market clearing condition with equations (1), (5) and (6). This implies that the equilibrium wage vector must satisfy

$$\sum_j \left[\frac{\sum_{s \in \mathcal{S}_i} \left(\frac{\tau_{ij,s} w_i}{\Psi_i(\Phi(\mathbf{w}, \mathbf{P}(\mathbf{w}|\boldsymbol{\tau})))} \right)^{-\epsilon_s}}{\sum_{o: s \in \mathcal{S}_o} \left(\frac{\tau_{oj,s} w_o}{\Psi_o(\Phi(\mathbf{w}, \mathbf{P}(\mathbf{w}|\boldsymbol{\tau})))} \right)^{-\epsilon_s}} \xi_{j,s} \right] (w_j \Phi_j(\mathbf{w}, \mathbf{P}(\mathbf{w}|\boldsymbol{\tau})) + T_j) = w_i \Phi_i(\mathbf{w}, \mathbf{P}(\mathbf{w}|\boldsymbol{\tau})) (w_j L_j).$$

C.1.2 Counterfactual analysis

We now extend the counterfactual analysis of Section 3. We present the derivations below and focus here on the main implications of trade imbalances for our results. Notice that the equilibrium system has similar structure to that of our baseline model with the only difference being the exogenous transfer T_j . Thus, we obtain a similar system to characterize changes in relative wages:

$$\bar{\gamma} \hat{\mathbf{w}} = \hat{\boldsymbol{\eta}} \quad (\text{C.2})$$

where

$$\hat{\boldsymbol{\eta}} \equiv \hat{\boldsymbol{\eta}}^R - \bar{\boldsymbol{\alpha}} \bar{\boldsymbol{\phi}}^p \hat{\boldsymbol{\eta}}^C \quad (\text{C.3})$$

and

$$\bar{\boldsymbol{\gamma}} \equiv \bar{\mathbf{I}} - (\bar{\mathbf{y}}\bar{\mathbf{t}} + \bar{\boldsymbol{\rho}} + \bar{\boldsymbol{\chi}}) + \bar{\boldsymbol{\alpha}} (\bar{\boldsymbol{\phi}}^w + \bar{\boldsymbol{\phi}}^p \bar{\mathbf{x}}') \quad (\text{C.4})$$

with $\bar{\alpha} \equiv (\bar{\mathbf{I}} - \bar{\mathbf{y}}\bar{\mathbf{t}} + \bar{\chi}\bar{\psi})\bar{\rho}$ and $\bar{\rho} \equiv (\bar{\mathbf{I}} + \bar{\phi}^p\bar{x}'\bar{\psi})^{-1}$. The matrix $\bar{\mathbf{t}}$ is the diagonal matrix of output-spending ratios, $t_{ii}^0 \equiv Y_i^0/E_i^0$. The elements of the matrix $\bar{\rho}$ measure the elasticity of i 's demand to changes in transfers to j' driven by wage changes in different markets j : $\rho_{ij} = \sum_{j'} y_{ij'}^0 (1 - t_{j'j'}^0) \tilde{y}_j^0$ where $\tilde{y}_j^0 \equiv Y_j^0 / \sum_{j'} Y_{j'}^0$ is j 's share in world output.

Compared to the expressions in the baseline model, we can see that the revenue shares in the multiplier $\bar{\alpha}$ and in the spatial links matrix $\bar{\gamma}$ are now multiplied by the imbalance ratio diagonal matrix $\bar{\mathbf{t}}$. This represents the change in other markets' spending, i.e. $y_{ij}^0 t_{jj}$, triggered by wages and trade costs changes holding constant transfers. The matrix $\bar{\rho}$, instead, is the change in spending coming from transfers, which may change because the world's average wage changes in response to the shock (relative to the numeraire $w_m \equiv 1$).

C.1.3 Derivation of equation (C.2)

We start by totally differentiating the market clearing equation:

$$\hat{w}_i + \hat{L}_i = \hat{\eta}_i^R + \sum_j \frac{x_{ij} E_j}{Y_i} \hat{E}_j + \sum_j \frac{\partial \log Y_i}{\partial \log p_j} \hat{p}_j$$

where $\hat{\eta}_i^R$ is defined in Proposition 2. Notice that (5) implies that $\hat{p}_i = \hat{w}_i - \sum_j \psi_{ij} \hat{L}_j$, and $E_i = Y_i + T_i$ implies that $\hat{E}_i = \frac{Y_i}{E_i}(\hat{w}_i + \hat{L}_i) + \frac{T_i}{E_i} \hat{T}_i$. Thus, in matrix form:

$$\hat{\mathbf{w}} + \hat{\mathbf{L}} = \hat{\boldsymbol{\eta}}^R + \bar{\mathbf{y}}\bar{\mathbf{t}} \left(\hat{\mathbf{w}} + \hat{\mathbf{L}} \right) + \bar{\boldsymbol{\rho}}\hat{\mathbf{w}} + \bar{\boldsymbol{\chi}} \left(\hat{\mathbf{w}} - \bar{\boldsymbol{\psi}}\hat{\mathbf{L}} \right). \quad (\text{C.5})$$

By rearranging this expression, we get

$$(\bar{\mathbf{I}} - \bar{\mathbf{y}}\bar{\mathbf{t}} - \bar{\boldsymbol{\rho}} - \bar{\boldsymbol{\chi}}) \hat{\mathbf{w}} + (\bar{\mathbf{I}} - \bar{\mathbf{y}}\bar{\mathbf{t}} + \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\psi}}) \hat{\mathbf{L}} = \hat{\boldsymbol{\eta}}^R$$

Applying the result in equation (21) into this expression, we get

$$(\bar{\mathbf{I}} - \bar{\mathbf{y}}\bar{\mathbf{t}} - \bar{\boldsymbol{\rho}} - \bar{\boldsymbol{\chi}}) \hat{\mathbf{w}} + (\bar{\mathbf{I}} - \bar{\mathbf{y}}\bar{\mathbf{t}} + \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\psi}}) \left(\bar{\boldsymbol{\rho}}\bar{\boldsymbol{\phi}}^w \hat{\mathbf{w}} + \bar{\boldsymbol{\rho}}\bar{\boldsymbol{\phi}}^p \left(\hat{\boldsymbol{\eta}}^C + \bar{\mathbf{x}}'\hat{\mathbf{w}} \right) \right) = \hat{\boldsymbol{\eta}}^R,$$

which implies that

$$\left[(\bar{\mathbf{I}} - \bar{\mathbf{y}}\bar{\mathbf{t}} - \bar{\boldsymbol{\rho}} - \bar{\boldsymbol{\chi}}) + (\bar{\mathbf{I}} - \bar{\mathbf{y}}\bar{\mathbf{t}} + \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\psi}}) \bar{\boldsymbol{\rho}} (\bar{\boldsymbol{\phi}}^w + \bar{\boldsymbol{\phi}}^p \bar{\mathbf{x}}) \right] \hat{\mathbf{w}} = \hat{\boldsymbol{\eta}}^R - (\bar{\mathbf{I}} - \bar{\mathbf{y}}\bar{\mathbf{t}} + \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\psi}}) \bar{\boldsymbol{\rho}}\bar{\boldsymbol{\phi}}^p \hat{\boldsymbol{\eta}}^C.$$

The definitions of $\bar{\alpha}$ and $\bar{\gamma}$ imply that this expression is equivalent to (C.2).

C.2 Model with Bilateral Migration

C.2.1 Environment

Labor Supply We assume that bilateral migration flows from i to j are given by the following function:

$$M_{ij} = \tilde{\Phi}_{ij}(\mathbf{w}, \mathbf{P}). \quad (\text{C.6})$$

As discussed in Section B, this general specification covers the environment in Bryan and Morten (2015) where individuals born in region i have heterogeneous region-specific efficiency and make a discrete choice of which region j to reside and work. We allow bilateral labor supply to be a function of the vector of wages and prices in all markets. In this case, spatial links can be summarized by the elasticity of the

bilateral labor supply to changes in wages and prices in different markets:

$$\tilde{\phi}_{ij,o}^w \equiv \frac{\partial \ln \tilde{\Phi}_{ij}(\mathbf{w}, \mathbf{P})}{\partial \ln w_o} \quad \text{and} \quad \tilde{\phi}_{ij,o}^p \equiv \frac{\partial \ln \tilde{\Phi}_{ij}(\mathbf{w}, \mathbf{P})}{\partial \ln P_o}.$$

In equilibrium, labor supply in region j is the sum of the labor supply originated in different markets:

$$L_j = \Phi_j(\mathbf{w}, \mathbf{P}) \equiv \sum_i \tilde{\Phi}_{ij}(\mathbf{w}, \mathbf{P}). \quad (\text{C.7})$$

Thus, spatial links in labor supply are determined by the elasticity structure of the bilateral labor supply function:

$$\phi_{jo}^w \equiv \frac{\partial \ln \Phi_j(\mathbf{w}, \mathbf{P})}{\partial \ln w_o} = \sum_i m_{ij} \tilde{\phi}_{ij,o}^w \quad \text{and} \quad \phi_{jo}^p \equiv \frac{\partial \ln \Phi_j(\mathbf{w}, \mathbf{P})}{\partial \ln P_o} = \sum_i m_{ij} \tilde{\phi}_{ij,o}^p \quad (\text{C.8})$$

where m_{ij} is the share of the labor force of j coming from origin market i .

Productivity and Trade Demand We assume that the production technology and demand for goods are identical to those in the baseline model, so that equations (3)–(8) still hold.

Equilibrium Conditional on the labor supply function in (C.7), the excess labor demand in (9) remains valid. The equilibrium wage vector then solves the excess demand system in (10).

C.2.2 Counterfactual analysis

Given that the equilibrium remains the same, all the results in Section 3 still hold for the labor supply elasticity matrices in (C.8). However, this extended version of the model yields predictions regarding changes in bilateral migration flows. Equation (C.6) immediately implies that

$$\hat{M}_{ij} = \sum_o \tilde{\phi}_{ij,o}^w \hat{w}_o + \sum_o \tilde{\phi}_{ij,o}^p \hat{P}_o,$$

where, as in the baseline model, $\hat{\mathbf{P}} = \hat{\eta}^C + \bar{\mathbf{x}}^{0'} (\hat{\mathbf{w}} - \bar{\psi} \hat{\mathbf{L}})$, $\hat{\mathbf{w}}$ is given by (17), and $\hat{\mathbf{L}}$ is given by (21).

C.3 Model with Multiple Labor Types

C.3.1 Environment

We assume that the production in region r and sector s of market i uses multiple types of workers, indexed with g . There are N markets and G groups. In this section, we use the notation $\tilde{\mathbf{x}}$ to indicate a stacked vector of size $M \times 1$, where $M = GN$, such that the first N rows are the variables for group $g = 1$, the rows from $N + 1$ to $2N$ are the variables for group $g = 2$, and so on. We use the notation $\tilde{\tilde{\mathbf{x}}}$ to indicate matrices of length M with the same stacked configuration.

Trade Demand. We maintain the same nested gravity trade demand of our baseline model such that the spending share of market j on goods produced in i is

$$x_{ij}(\mathbf{p}|\boldsymbol{\tau}) = \sum_{s \in \mathcal{S}_i} \frac{(\tau_{ij,sp_i})^{-\epsilon_s}}{\sum_{o:s \in \mathcal{S}_o} (\tau_{oj,sp_o})^{-\epsilon_s}} \xi_{j,s}, \quad (\text{C.9})$$

and the price index in market j is

$$P_j(\mathbf{p}|\boldsymbol{\tau}) = \prod_s \left[\sum_{o:s \in \mathcal{S}_o} (\tau_{oj,s} p_o)^{-\epsilon_s} \right]^{\frac{\xi_{j,s}}{-\epsilon_s}}. \quad (\text{C.10})$$

This assumption effectively imposes that all worker groups in market i have identical preferences, so that they have the same spending shares and price indices in a given market. Thus, the stacked vector of price indices $\tilde{\mathbf{P}}$ has identical entries for all groups in the same market.

Production. We assume that the production function is a CES across different worker groups:

$$Q_{r,s} = \Psi_i(\tilde{\mathbf{L}}) \left(\sum_g \vartheta_{ig}^{\frac{1}{\rho}} (L_{rsq})^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}},$$

where ρ is the elasticity of substitution between labor types. The cost minimization problem of the representative firm implies that the zero profit condition is

$$p_i = \frac{\left[\sum_g (w_{ig})^{1-\rho} \right]^{\frac{1}{1-\rho}}}{\Psi_i(\tilde{\mathbf{L}})}. \quad (\text{C.11})$$

Notice that we have now labor productivity links that depend on group-level employment. However, the production function imposes that the effect is the same on all worker groups employed in a region. Accordingly, the matrix summarize spatial links in labor productivity has dimension $N \times M$:

$$\psi_{ijg} \equiv \frac{\partial \ln \Psi_i(\tilde{\mathbf{L}})}{\partial \ln L_{jg}}.$$

Labor Supply. The labor supply function of each worker type g is

$$L_{ig} = \Phi_{ig}(\tilde{\mathbf{w}}, \tilde{\mathbf{P}}), \quad (\text{C.12})$$

where $\Phi_{ig}(\cdot)$ is strictly positive, differentiable, bounded from above, and homogeneous of degree zero in $(\tilde{\mathbf{w}}, \tilde{\mathbf{P}})$. We use again the matrices of labor supply elasticities to changes in wages and prices to summarize the economy's spatial links in labor supply,

$$\phi_{ig,jb}^w \equiv \frac{\partial \ln \Phi_{ig}(\tilde{\mathbf{w}}, \tilde{\mathbf{P}})}{\partial \ln w_{jb}} \quad \text{and} \quad \phi_{igj}^p \equiv \frac{\partial \ln \Phi_{ig}(\tilde{\mathbf{w}}, \tilde{\mathbf{P}})}{\partial \ln P_j}.$$

We will use the notation $\tilde{\tilde{\phi}}^w$ to denote the matrix $M \times M$ of labor supply elasticity to wages. We allow the supply of a worker group to depend on relative wages across groups. In addition, we denote $\tilde{\tilde{\phi}}^p$ as the $M \times N$ matrix of labor supply elasticity to changes in the price index. It has N columns because the price index is the same for all groups in the same market.

Market Clearing. Since all sectors have the same CES labor aggregator across worker types, the spending share on group g for each sector s is

$$\mu_{ig} \equiv \frac{\vartheta_{ig} (w_{ig})^{1-\rho}}{\sum_b \vartheta_{ib} (w_{ib})^{1-\rho}}. \quad (\text{C.13})$$

Thus, the market clearing condition for each group is

$$w_{ig}L_{ig} = \mu_{ig}Y_i(\mathbf{p}|\boldsymbol{\tau}) = \mu_{ig} \sum_j x_{ij}(\mathbf{p}|\boldsymbol{\tau})E_j. \quad (\text{C.14})$$

Finally, we assume again trade balance so that total expenditures in market j are also equal to the sum of the labor income of all groups in that markets:

$$E_j = \sum_g w_{jg}L_{jg}.$$

C.3.2 Counterfactual analysis

We now extend the counterfactual analysis of Section 3. We present the derivations below and focus here on the main implications of trade imbalances for our results. In this case, we obtain a similar system determining relative wage changes across both groups and markets:

$$\bar{\gamma}\hat{\mathbf{w}} = \hat{\mathbf{w}} \quad (\text{C.15})$$

where

$$\begin{aligned} \hat{\boldsymbol{\eta}} &\equiv \hat{\boldsymbol{\eta}}^R - \bar{\boldsymbol{\alpha}}\bar{\boldsymbol{\phi}}^p\hat{\boldsymbol{\eta}}^C \\ \bar{\gamma} &\equiv \rho\bar{\mathbf{I}} - ((\rho - 1)\bar{\mathbf{I}} + \bar{\mathbf{y}} + \bar{\boldsymbol{\chi}})\bar{\boldsymbol{\mu}} + \bar{\boldsymbol{\alpha}}\left(\bar{\boldsymbol{\phi}}^w + \bar{\boldsymbol{\phi}}^p\bar{\mathbf{x}}'\bar{\boldsymbol{\mu}}\right) \end{aligned}$$

with $\bar{\boldsymbol{\alpha}} \equiv \left(\bar{\mathbf{I}} - \bar{\mathbf{y}}\bar{\boldsymbol{\mu}} + \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\psi}}\right)\left(\bar{\mathbf{I}} + \bar{\boldsymbol{\phi}}^p\bar{\mathbf{x}}'\bar{\boldsymbol{\psi}}\right)^{-1}$.

It is important to notice that the economy with multiple labor groups has a similar shift in excess labor demand. Here, the $\hat{\boldsymbol{\eta}}^R$ is just the stacked vector of revenue shifts with the entry of group g in market i given by the same $\hat{\eta}_i^R$ defined in Proposition 2 and $\hat{\boldsymbol{\eta}}^C$ is the same stacked vector of consumption cost shifts $\hat{\eta}_i^C$ defined in Proposition 2.

Relative to the baseline model, the economy with multiple labor groups entails two modifications. First, the substitution effect in the spatial links matrix also accounts for the substitution across worker groups in the same market. This is captured by the terms multiplied by the elasticity of substitution ρ . Second, changes in production costs and market size also account for the initial share of each group in the market's spending, μ_{jg} . This is captured by the $N \times M$ matrix $\bar{\boldsymbol{\mu}}$ whose row j has entries μ_{jg} for column in the j -th entry for the sub-vector of group g and zero in all other entries.

C.3.3 Derivation of equation (C.15)

Using the definition of $\hat{\eta}_i^R$ in Proposition 2, the total differentiation of the market clearing condition (C.14) implies

$$\hat{w}_{ig} + \hat{L}_{ig} - \hat{\mu}_{ig} = \hat{\eta}_i^R + \sum_j \frac{x_{ij}E_j}{Y_i}\hat{E}_j + \sum_j \frac{\partial \log Y_i}{\partial \log p_j}\hat{p}_j. \quad (\text{C.16})$$

From (C.13), the change in the labor share of g is

$$\hat{\mu}_{ig} = (1 - \rho)\hat{w}_{ig} - (1 - \rho)\sum_b \mu_{ib}\hat{w}_{ib}, \quad (\text{C.17})$$

and, from (C.11), the change in production cost of i is

$$\hat{p}_i = \sum_g \mu_{ig} \hat{w}_{ig} - \sum_{j,g} \psi_{ijg} \hat{L}_{jg}. \quad (\text{C.18})$$

Plugging equation (C.17) and (C.18) into (C.16), we obtain

$$\begin{aligned} \hat{w}_{ig} + \hat{L}_{ig} - (1 - \rho) (\hat{w}_{ig} - \sum_b \mu_{ib} \hat{w}_{ib}) &= \hat{\eta}_i^R + \sum_j y_{ij} \sum_b \mu_{jb} (\hat{w}_{jb} + \hat{L}_{jb}) \\ &+ \sum_j \chi_{ij} \left(\sum_b \mu_{jb} \hat{w}_{jb} - \sum_{j',b'} \psi_{jj'b'} \hat{L}_{j'b'} \right). \end{aligned}$$

We can write this system in matrix form:

$$\hat{\mathbf{w}} + \hat{\mathbf{L}} - (1 - \rho) (\hat{\mathbf{w}} - \bar{\boldsymbol{\mu}} \hat{\mathbf{w}}) = \hat{\boldsymbol{\eta}}^R + \bar{\mathbf{y}} \bar{\boldsymbol{\mu}} (\hat{\mathbf{w}} + \hat{\mathbf{L}}) + \bar{\boldsymbol{\chi}} (\bar{\boldsymbol{\mu}} \hat{\mathbf{w}} - \bar{\boldsymbol{\psi}} \hat{\mathbf{L}}),$$

where (i) $\hat{\boldsymbol{\eta}}^R$ is the $M \times 1$ vector with $\hat{\eta}_i^R$ for all groups g in market i , (ii) $\bar{\boldsymbol{\mu}}$ is the $N \times M$ matrix where row j has entries μ_{jg} for column in the j -th entry for the sub-vector of group g and zero in all other entries, and (iii) $\bar{\mathbf{y}}$ and $\bar{\boldsymbol{\chi}}$ are the $M \times N$ matrices where the row of group g in market i has entries $\{y_{ij}\}_{j=1}^N$ and $\{\chi_{ij}\}_{j=1}^N$, respectively. By rearranging this expression, we get that

$$\left(\bar{\mathbf{I}} - \bar{\mathbf{y}} \bar{\boldsymbol{\mu}} + \bar{\boldsymbol{\chi}} \bar{\boldsymbol{\psi}} \right) \hat{\mathbf{L}} + \left(\bar{\mathbf{I}} - (1 - \rho) (\bar{\mathbf{I}} - \bar{\boldsymbol{\mu}}) - \bar{\mathbf{y}} \bar{\boldsymbol{\mu}} - \bar{\boldsymbol{\chi}} \bar{\boldsymbol{\mu}} \right) \hat{\mathbf{w}} = \hat{\boldsymbol{\eta}}^R \quad (\text{C.19})$$

We then turn to the change in employment implied by the shock. Using the definition of $\hat{\eta}_i^C$ in Proposition 2, the total differentiation of the labor supply equation in (C.12) and the price index in (C.10) imply that

$$\hat{L}_{ig} = \sum_{j,b} \phi_{igjb}^w \hat{w}_{jg} + \sum_j \phi_{igj}^p \left(\hat{\eta}_j^C + \sum_o x_{oj} \hat{p}_o \right).$$

Using the expression for production cost change in (C.18), this expression can be written as

$$\hat{L}_{ig} = \sum_{j,b} \phi_{igjb}^w \hat{w}_{jg} + \sum_j \phi_{igj}^p \left(\hat{\eta}_j^C + \sum_o x_{oj} \left(\sum_b \mu_{ob} \hat{w}_{ob} - \sum_{j',b} \psi_{oj'b} \hat{L}_{j'b} \right) \right).$$

Again, using the same definitions, we write this system in matrix form:

$$\hat{\mathbf{L}} = \bar{\boldsymbol{\phi}}^w \hat{\mathbf{w}} + \bar{\boldsymbol{\phi}}^p \left(\hat{\boldsymbol{\eta}}^C + \bar{\mathbf{x}}' (\bar{\boldsymbol{\mu}} \hat{\mathbf{w}} - \bar{\boldsymbol{\psi}} \hat{\mathbf{L}}) \right)$$

where, as in the baseline model, $\hat{\boldsymbol{\eta}}^C$ is the $N \times 1$ stacked matrix of consumption cost shifts across markets and $\bar{\mathbf{x}}$ is $N \times N$ matrix of spending shares. By rearranging this expression, we obtain

$$\tilde{\mathbf{L}} = \bar{\boldsymbol{\rho}} \bar{\boldsymbol{\phi}}^w \hat{\mathbf{w}} + \bar{\boldsymbol{\rho}} \bar{\boldsymbol{\phi}}^p \left(\hat{\boldsymbol{\eta}}^C + \bar{\mathbf{x}}' \bar{\boldsymbol{\mu}} \hat{\mathbf{w}} \right) \quad (\text{C.20})$$

where $\bar{\boldsymbol{\rho}} \equiv \left(\bar{\mathbf{I}} + \bar{\boldsymbol{\phi}}^p \bar{\mathbf{x}}' \bar{\boldsymbol{\psi}} \right)^{-1}$ is the $M \times M$ matrix that captures the multiplier employment effect of endogenous changes in productivity, prices, and labor supply.

The combination of the employment change in (C.20) and the market clearing condition in (C.19) implies that

$$\left(\bar{\mathbf{I}} - \bar{\mathbf{y}} \bar{\boldsymbol{\mu}} + \bar{\boldsymbol{\chi}} \bar{\boldsymbol{\psi}} \right) \left(\bar{\boldsymbol{\rho}} \bar{\boldsymbol{\phi}}^w \hat{\mathbf{w}} + \bar{\boldsymbol{\rho}} \bar{\boldsymbol{\phi}}^p \left(\hat{\boldsymbol{\eta}}^C + \bar{\mathbf{x}}' \bar{\boldsymbol{\mu}} \hat{\mathbf{w}} \right) \right) + \left(\rho \bar{\mathbf{I}} + (1 - \rho) \bar{\boldsymbol{\mu}} - \bar{\mathbf{y}} \bar{\boldsymbol{\mu}} - \bar{\boldsymbol{\chi}} \bar{\boldsymbol{\mu}} \right) \hat{\mathbf{w}} = \hat{\boldsymbol{\eta}}^R$$

Let us define $\bar{\alpha} \equiv \left(\bar{\mathbf{I}} - \bar{\mathbf{y}}\bar{\boldsymbol{\mu}} + \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\psi}} \right) \bar{\boldsymbol{\rho}}$. By rearranging this expression, we get that

$$\left[\rho \bar{\mathbf{I}} - ((\rho - 1)\bar{\mathbf{I}} + \bar{\mathbf{y}} + \bar{\boldsymbol{\chi}}) \bar{\boldsymbol{\mu}} + \bar{\alpha} \left(\bar{\boldsymbol{\phi}}^w + \bar{\boldsymbol{\phi}}^p \bar{\mathbf{x}}' \bar{\boldsymbol{\mu}} \right) \right] \tilde{\mathbf{w}} = \tilde{\boldsymbol{\eta}}^R - \bar{\alpha} \bar{\boldsymbol{\phi}}^p \hat{\boldsymbol{\eta}}^C,$$

which gives equation (C.15).

C.4 Model with Generalized Demand

We now generalize the nested CES trade demand in the model of Section 2. We follow Adao et al. (2017) by considering a general class of one-factor economies without restrictions on preferences and technologies. However, in contrast to Adao et al. (2017), we also allow for endogenous labor supply and labor productivity in each market. As in Adao et al. (2017), it is useful to further refine the definition of a market to include regions and sectors whose trade cost shocks change proportionally. In this case, we can completely omit the sector subscript and think of a market i as a collection of regions and sectors with identical changes in bilateral trade shifts, $\hat{\tau}_{ij}$.

In order to model this general environment, we consider again a set of aggregate functions that summarize the implications of this alternative production structure for trade demand. We establish below the equivalence of this aggregate specification and a general one-factor Ricardian model with external economies of scale.

C.4.1 Environment

Representative Household. We assume that each country has a representative household that decides the allocation of consumption and employment across markets. We denote the representative household's utility function as

$$U_c(\mathbf{C}, \mathbf{L}),$$

where $\mathbf{C} \equiv \{C_i\}_i$ and $\mathbf{L} \equiv \{L_i\}_i$ are respectively vectors of consumption and labor supply in all markets. We assume that $U_c(\cdot)$ is twice differentiable, increasing in \mathbf{C} , and quasi-concave in (\mathbf{C}, \mathbf{L}) . We also assume that C_j is an index that aggregates quantities consumed of the differentiated goods produced in different origin markets,

$$C_j \equiv V_j(\mathbf{c}_j),$$

where $\mathbf{c}_j \equiv \{c_{ij}\}_i$ with c_{ij} denoting the consumption in market j of the good produced in market i . We assume that the function $V_j(\cdot)$ is twice differentiable, increasing, and quasi-concave in all arguments. Importantly, we also restrict $V_j(\cdot)$ to be homogeneous of degree one, so that we can separate the problem of allocating spending shares across origin markets from the problem of determining labor supply across markets in the country.

We only allow for exogenous transfers across markets, so that the representative household faces the following budget constraint:

$$\sum_j p_{ij} c_{ij} = w_j L_j + T_j,$$

where w_j is the wage, P_j is the price of the homogeneous good, and T_j is an exogenous transfer.

Trade demand. The homogeneity of $V_j(\cdot)$ implies that, conditional on prices, the solution of the cost minimization problem yields the price index in market j :

$$P_j = P_j(\mathbf{p}_j) \equiv \min_{\mathbf{c}_j} \sum_o p_{oj} c_{oj} \quad \text{s.t.} \quad V_j(\mathbf{c}_j) = 1, \quad (\text{C.21})$$

where p_{ij} is the price of the good produced in market i sold in market j .

This problem yields a spending share on goods from origin i given by

$$x_{ij} = X_{ij}(\mathbf{p}_j). \quad (\text{C.22})$$

The price index and spending share functions inherit the usual properties of demand implied by utility maximization. The price index $P_j(\cdot)$ is homogeneous of degree one, concave, and differentiable. In addition, $X_{ij}(\cdot)$ is a convex set, with a single element if $V_j(\cdot)$ is strictly quasi-concave.

Labor Supply. We consider a competitive environment where, in deciding consumption and labor supply, the representative agent takes as given prices and wages. Given the solution of the spending minimization problem above, the budget constraint is $P_j C_j = w_j L_j + T_j$. Thus, the utility maximization problem yields the labor supply in market j :

$$L_j = \Phi_j(\mathbf{w}, \mathbf{P}). \quad (\text{C.23})$$

Production. In each market, there exists a representative firm that operates under perfect competition. Production requires only labor and it is subject to external economies of scale. Market i 's production function is

$$Y_i = \Psi_i(\mathbf{L}) L_i,$$

where $\Psi_i(\cdot)$ is a strictly positive real function. As in Section 2, the profit maximization problem implies that

$$p_i = \frac{w_i}{\Psi_i(\mathbf{L})}. \quad (\text{C.24})$$

We also impose that there are iceberg trade costs to ship goods between markets, so that

$$p_{ij} = \tau_{ij} p_i. \quad (\text{C.25})$$

Market Clearing. To close the model, we specify the labor market clearing condition. In each market,

$$w_i L_i = \sum_j x_{ij} E_j. \quad (\text{C.26})$$

Equilibrium. From equations (C.22) and (C.25), we can define the revenue of market i as

$$Y_i(\mathbf{p}, \mathbf{E}|\boldsymbol{\tau}) \equiv \sum_j X_{ij}(\{\tau_{ij} p_i\}_j) E_j.$$

This is the main change relative to our baseline model. The gravity structure of our model yields a specific functional form for the revenue function. Instead, the general preference structure $V_j(\cdot)$ implies that revenue depends on the demand function $X_{ij}(\cdot)$. The spatial links in trade demand are still summarized by the elasticity of the revenue function to changes in production costs, which now has the following general form

$$\chi_{ij} \equiv \frac{\partial \ln Y_i(\mathbf{p}, \mathbf{E}|\boldsymbol{\tau})}{\partial \ln p_j} = \sum_d y_{id} \lambda_{idj}, \quad (\text{C.27})$$

where $\lambda_{idj} \equiv \frac{\partial \log X_{id}}{\partial \log p_{dj}}$ is a general specification of the trade elasticity.

As in Section 2, to define the equilibrium of the economy, we need to solve for the price index as a function of the wage vector \mathbf{w} , $P_j = P_j(\mathbf{w}|\bar{\boldsymbol{\tau}})$. From equations (8), (C.23), (C.24) and (C.25), $P_j \in P_j(\mathbf{w}|\bar{\boldsymbol{\tau}})$ if, and only if,

$$P_j = P_j \left(\left\{ \tau_{ij} \frac{w_i}{\Psi_i(\boldsymbol{\Phi}(\mathbf{w}, \mathbf{P}))} \right\}_j \right) \quad \forall j. \quad (\text{C.28})$$

Finally, we can write the market clearing condition in terms of the wage vector. Using equations (C.23) (C.24) and (C.25), the market clearing condition in (C.26) is

$$\sum_j X_{ij} \left(\left\{ \tau_{ij} \frac{w_i}{\Psi_i(\boldsymbol{\Phi}(\mathbf{w}, \mathbf{P}(\mathbf{w}|\bar{\boldsymbol{\tau}})))} \right\}_j \right) (w_j \Phi_j(\mathbf{w}, \mathbf{P}(\mathbf{w}|\bar{\boldsymbol{\tau}})) + T_j) = w_i \Phi_i(\mathbf{w}, \mathbf{P}(\mathbf{w}|\bar{\boldsymbol{\tau}})). \quad (\text{C.29})$$

Given the normalization that $w_m \equiv 1$ for an arbitrary market m , equilibrium wage vector \mathbf{w} must satisfy the market clearing condition in (C.29) for all i .

C.4.2 Counterfactual analysis

We now extend the counterfactual analysis of Section 3 for changes in the bilateral trade shifts τ_{ij} . To this end, notice that the equilibrium of the economy has a similar structure as that of the baseline model in Section 2. The main difference is that it entails general functions for the price index in (C.28) and the trade demand in (C.29). Accordingly, we now show that the results of Section 3 still hold with modified definitions for the revenue shift and the trade elasticity matrix.

In this case, the revenue shift is

$$\hat{\eta}_i^R(\hat{\boldsymbol{\tau}}) \equiv \sum_{j,o} \frac{\partial \ln Y_i(\mathbf{p}^0, \mathbf{E}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{oj}} \hat{\tau}_{oj} = \sum_{o,j} y_{ij}^0 \lambda_{ij o} \hat{\tau}_{oj}. \quad (\text{C.30})$$

The more general demand function $X_{ij}(\mathbf{p}_j)$ in $Y_i(\mathbf{p}, \mathbf{E}|\boldsymbol{\tau})$ yields a different functional form for the revenue shift. In this case, the revenue shift is a function of the sensitivity of the demand for goods from i in different markets when the cost of exporting to those markets change. From Shepard's Lemma, the price index in equation (C.21) implies

$$\hat{\mathbf{P}} = \hat{\boldsymbol{\eta}}^C + \bar{\mathbf{x}}' \hat{\mathbf{p}},$$

where $\hat{\boldsymbol{\eta}}^C$ is defined in Proposition 2. Note that $\hat{\boldsymbol{\eta}}^C$ does not depend on the assumptions on the demand system because it follows directly from applying the envelope theorem to the minimization problem in (C.21). Conditional on the new definitions in (C.27) and (C.30), we have exactly the same set of equations as in the baseline model:

$$\bar{\boldsymbol{\gamma}} \hat{\mathbf{w}} = \hat{\boldsymbol{\eta}} \quad (\text{C.31})$$

where

$$\hat{\boldsymbol{\eta}} \equiv \hat{\boldsymbol{\eta}}^R - \bar{\boldsymbol{\alpha}} \bar{\boldsymbol{\phi}}^p \hat{\boldsymbol{\eta}}^C \quad (\text{C.32})$$

and

$$\bar{\boldsymbol{\gamma}} \equiv \bar{\mathbf{I}} - (\bar{\mathbf{y}}\bar{\mathbf{u}} + \bar{\boldsymbol{\rho}} + \bar{\boldsymbol{\chi}}) + \bar{\boldsymbol{\alpha}} (\bar{\boldsymbol{\phi}}^w + \bar{\boldsymbol{\phi}}^p \bar{\mathbf{x}}') \quad (\text{C.33})$$

with $\bar{\boldsymbol{\alpha}} \equiv (\bar{\mathbf{I}} - \bar{\mathbf{y}}\bar{\mathbf{u}} + \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\psi}}) \bar{\boldsymbol{\rho}}$ and $\bar{\boldsymbol{\rho}} \equiv (\bar{\mathbf{I}} + \bar{\boldsymbol{\phi}}^p \bar{\mathbf{x}}' \bar{\boldsymbol{\psi}})^{-1}$.

C.4.3 Equivalence with a general one-factor Ricardian model with external economies of scale

For completeness, we now show that the environment above is observationally equivalent to a general one-factor Ricardian model with external economies of scale.

One-factor Ricardian model with external economies of scale. We consider an economy with an arbitrary number of goods indexed by s . We assume that each country has a representative agent with preferences for consumption and labor supply in different markets, with utility function given by

$$U_c \left(\{C_j^N\}_j, \{L_j^N\}_j \right) \quad \text{such that} \quad C_j^N \equiv V^N \left(\{c_{ij,s}^N\}_{i,s} \right).$$

where $V^N(\cdot)$ is twice differentiable, quasi-concave, homothetic, and increasing in all arguments. Notice that $V^N(\cdot)$ allows for the possibility that goods from different origins are imperfect substitutes.

Let $p_{ij,s}$ be the price of good s from i in market j . The representative household's budget constraint is

$$\sum_i \sum_s p_{ij,s}^N c_{ij,s}^N = w_j^N L_j^N + T_j.$$

There are many perfectly competitive firms supplying each good in any market. The production technology uses only labor and entails external economies of scale at the market level. In particular, the technology of producing good s in market i and delivering to j is given by

$$Y_{ij,s}^N = \Psi_i(\mathbf{L}^N) \frac{L_{ij,s}^N}{\tau_{ij} \alpha_{ij,s}^N},$$

where $\alpha_{ij,s}^N$ is good-specific productivity of producing in i and delivering in j . We restrict trade costs to be identical to all goods being traded between markets i and j : $\tau_{ij,s} = \tau_{ij}$ for all s .

Equilibrium. We use the fact that $V^N(\cdot)$ is homothetic to derive the price index in market j :

$$P_j^N = P_j^N \left(\{p_{oj,k}^N\}_{o,k} \right) \equiv \min_{\{c_{k,oj}\}_{k,o}} \sum_{k,o} p_{oj,k}^N c_{oj,k} \quad \text{s.t.} \quad V^N \left(\{c_{oj,k}\}_{o,k} \right) \geq 1 \quad (\text{C.34})$$

where the associated spending share on good s from i is

$$x_{ij,s}^N = X_{ij,s}^N \left(\{p_{oj,k}^N\}_{o,k} \right). \quad (\text{C.35})$$

Conditional on prices, the representative household solves the utility maximization problem that yields the labor supply in market j :

$$L_j^N = \Phi_j(\mathbf{w}^N, \mathbf{P}^N). \quad (\text{C.36})$$

Profit maximization implies that

$$p_{ij,s}^N = \tau_{ij} p_i^N \alpha_{ij,s}^N \quad (\text{C.37})$$

where

$$p_i^N = \frac{w_i^N}{\Psi_i(\mathbf{L}^N)}. \quad (\text{C.38})$$

Finally, the labor market clearing condition is

$$w_i^N L_i^N = \sum_j \sum_s x_{ij,s}^N (w_j^N L_j^N). \quad (\text{C.39})$$

The equilibrium can be written as $\{p_i^N, w_i^N, L_i^N, P_i^N\}_i$ solving (8)–(C.39) with $\Phi_j(\cdot)$, $\Psi_j(\cdot)$, and

$$X_{ij}^N(\{\tau_{oj} p_o^N\}_o) \equiv \left\{ x_{ij}^N = \sum_s x_{ij,s}^N : x_{ij,s}^N = X_{ij,s}^N(\{\tau_{oj} p_o^N \alpha_{oj,k}^N\}_{o,k}) \right\} \quad (\text{C.40})$$

such that

$$P_i^N(\{\tau_{oj} p_o^N\}_o) = P_i^N(\{\tau_{oj} p_o^N \alpha_{oj,k}^N\}_{o,k}).$$

Equivalence with environment in Section C.4.1. We now construct an equivalent economy whose equilibrium vector satisfies a condition identical to (C.29). Production in market i is

$$Q_i = \Psi_i(L) L_i.$$

To deliver in market j , producers of i face iceberg trade costs so that

$$p_{ij} = \tau_{ij} p_i.$$

In addition, each country has a representative agent with preferences for consumption and labor supply in different markets, with utility function given by

$$U_c(\{C_j^N\}_j, \{L_j^N\}_j).$$

with

$$V_j(\{c_{ij}\}_i) \equiv \max_{\{c_{ij,s}\}_{i,s}} V^N(\{c_{ij,s}\}_{i,s}) \quad \text{s.t.} \quad \sum_s \alpha_{ij,s}^N c_{ij,s} = c_{ij},$$

and associated spending shares given by

$$x_{ij} \in X_{ij}(\{\tau_{oj} p_o\}_o).$$

It is straight forward to see that this alternative economy has the same structure as the economy introduced in Section C.4.1. Thus, to formally establish the equivalence with the one-fact Ricardian economy above, it is sufficient to show that

$$X_{ij}(\{\tau_{oj} p_o\}_o) = X_{ij}^N(\{\tau_{oj} p_o\}_o) \quad \forall \{\tau_{oj} p_o\}_o, \quad (\text{C.41})$$

where $X_{ij}^N(\cdot)$ is the function defined in (C.40).

Intuitively, the preference structure above implies that, if the representative household acquires c_{ij} units of i 's composite good for j 's consumption, then it optimally allocates the composite good into the production of different goods, given the exogenous weights $\alpha_{z,ij}^N$ that are now embedded into the representative agent's preferences. Since the relative price of goods in market i only depends on $\alpha_{z,ij}^N$, this decision yields allocations that are identical to those in the competitive equilibrium of the decentralized economy.

First, we show that $x_{ij} \in X_{ij}(\{\tau_{oj} p_o\}_o) \implies \exists x_{ij,s}^N \in X_{ij,s}^N(\{\tau_{oj} p_o \alpha_{oj,k}^N\}_{o,k})$ with $x_{ij} = \sum_s x_{ij,s}^N$. Let $\{c_{z,ij}\}_{z,i}$ be the solution of the good allocation problem in the definition of $V_j(\{c_{ij}\})$. We proceed

by contradiction to show that $\{c_{ij,s}\}_{i,s}$ implies spending shares, $\{x_{ij,s}\}_{i,s} = \left\{ \tau_{oj} p_o \alpha_{ij,s}^N c_{ij,s} \right\}_{i,s}$, such that $x_{z,ij} \in X_{ij,s}^N \left(\left\{ \tau_{oj} p_o \alpha_{oj,k}^N \right\}_{o,k} \right)$. Suppose there exists a feasible allocation $\{c_{ij,s}^N\}_{i,s}$ such that

$$V^N \left(\{c_{ij,s}^N\}_{i,s} \right) > V^N \left(\{c_{ij,s}\}_{i,s} \right) \quad \text{and} \quad \sum_i \sum_s \tau_{ij} p_i \alpha_{ij,s}^N c_{ij,s}^N \leq 1. \quad (\text{C.42})$$

Notice that $\sum_i \sum_s \tau_{ij} p_i \alpha_{ij,s}^N c_{ij,s}^N \leq 1$, which implies that the allocation $c_{ij}^N \equiv \sum_z \alpha_{ij,s}^N c_{ij,s}^N$ is feasible in the equivalent economy. Thus,

$$V^N \left(\{c_{ij,s}\}_{i,s} \right) = V_j \left(\{c_{ij}\} \right) \geq V_j \left(\{c_{ij}^N\} \right) \geq V^N \left(\{c_{ij,s}^N\}_{i,s} \right),$$

which is a contradiction of inequality (C.42).

Second, we show that $x_{ij} = \sum_s x_{ij,s}^N$, with $x_{ij,s}^N \in X_{ij,s}^N \left(\left\{ \tau_{oj} p_o \alpha_{oj,k}^N \right\}_{o,k} \right)$, and $c_{ij}^N = \sum_z \alpha_{ij,s}^N c_{ij,s}^N$ imply that $x_{ij} \in X_{ij} \left(\{ \tau_{oj} p_o \}_o \right)$. We start with $c_{ij}^N = \sum_z \alpha_{ij,s}^N c_{ij,s}^N$ implied by the solution of the consumer's problem in the one-factor Ricardian economy. We proceed by contradiction to show that $\{c_{ij}^N\}_i$ is optimal in the equivalent economy given prices $\{\tau_{ij} p_i\}_i$. Suppose there exists a feasible allocation $\{c_{ij}\}_i$ in the equivalent economy such that

$$V_j \left(\{c_{ij}\} \right) > V_j \left(\{c_{ij}^N\} \right) \quad \text{and} \quad \sum_i p_{ij} c_{ij} \leq \sum_i p_{ij} c_{ij}^N = 1.$$

Let $\{c_{z,ij}\}_{z,i}$ be the be the solution of the good allocation problem in the definition of $V_j \left(\{c_{ij}\} \right)$. Thus,

$$\sum_i \tau_{ij} p_i \sum_z \alpha_{ij,s}^N c_{ij,s}^N = \sum_i \tau_{ij} p_i c_{ij} \leq 1$$

and, by revealed preference,

$$V_j \left(\{c_{ij}^N\} \right) \geq V^N \left(\{c_{ij,s}\}_{i,s} \right) \geq V^N \left(\{c_{ij,s}\}_{i,s} \right) = V_j \left(\{c_{ij}\} \right).$$

This is a contradiction, which establishes the result.

C.5 Model with Intermediate Goods in Production

C.5.1 Environment

As in our baseline model, we assume that all producers in a market face a single wage rate. To simplify exposition, we now assume that all sectors are present in a market, so that markets can be interpreted as geographic units where labor is perfectly mobile across sectors. We use S to denote the number of sectors and N to denote the number of markets.

Labor Supply. As in the baseline model, the labor supply in market i is given by equation (1).

Production. Following Caliendo and Parro (2015), we assume a Cobb-Douglas production function between labor and an intermediate inputs aggregator in sector s of market i :

$$Q_{i,s} = \Psi_i(\mathbf{L}) \left(\frac{L_{i,s}}{\varpi_{i,s}} \right)^{\varpi_{i,s}} \left(\frac{M_{i,s}}{1 - \varpi_{i,s}} \right)^{1 - \varpi_{i,s}}, \quad \text{such that} \quad M_{i,s} = \Pi_k \left(\frac{Q_{i,ks}}{\theta_{i,ks}} \right)^{\theta_{i,ks}}$$

with $\sum_k \theta_{i,ks} = 1$. In each sector s , the demand for inputs from different origin markets is given by the following constant elasticity function:

$$Q_{i,ks} = \left[\sum_j (Q_{ji,ks})^{\frac{\epsilon_k}{1+\epsilon_k}} \right]^{\frac{1+\epsilon_k}{\epsilon_k}}$$

where $Q_{ji,ks}$ is the good produced by sector k in market j consumed in sector s of market i .

Input Trade Demand. The cost minimization problem of the representative firm implies that the zero profit condition is

$$p_{ij,s} = \frac{\tau_{ij,s} p_{i,s}}{\Psi_i(\mathbf{L})} \quad (\text{C.43})$$

where

$$p_{i,s} = (w_i)^{\varpi_{i,s}} (P_{i,s}^M)^{1-\varpi_{i,s}} \quad (\text{C.44})$$

and

$$P_{i,s}^M = \prod_k \left[\sum_o \left(\frac{\tau_{oi,k} p_{o,k}}{\Psi_o(\mathbf{L})} \right)^{-\epsilon_k} \right]^{\frac{\theta_{i,ks}}{-\epsilon_k}}. \quad (\text{C.45})$$

The cost minimization problem above implies that the share of spending on sector k of market j by sector s of market i is

$$x_{ji,ks}^M = x_{ji,k} \theta_{i,ks} (1 - \varpi_{i,s})$$

where

$$x_{ji,k} = \frac{\left(\frac{\tau_{ji,k} p_{j,k}}{\Psi_j(\mathbf{L})} \right)^{-\epsilon_k}}{\sum_o \left(\frac{\tau_{oi,k} p_{o,k}}{\Psi_o(\mathbf{L})} \right)^{-\epsilon_k}}. \quad (\text{C.46})$$

Final Trade Demand. As in the baseline model, we assume that final consumption follows a nested gravity structure where the share of j 's final spending on sector k of market i is

$$x_{ji,k}^C = x_{ji,k} \xi_{i,k},$$

where $x_{ji,k}$ is the share of j 's spending in sector k on goods from i (as defined above). The shares $x_{ji,k}$ are the same in both final and intermediate spending shares because the elasticity of substitution across goods of different origins is the same as the one in the input demand. Caliendo and Parro (2015) introduce this assumption to circumvent the need of separately measuring trade flows for final and intermediate uses.

As in the baseline model, the consumption price index in market j is

$$P_j = \prod_s \left[\sum_{o:s \in \mathcal{S}_o} \left(\frac{\tau_{oj,s} p_{o,s}}{\Psi_o(\mathbf{L})} \right)^{-\epsilon_s} \right]^{\frac{\xi_{j,s}}{-\epsilon_s}}. \quad (\text{C.47})$$

Market clearing. The total spending by market i on sector k of market j is

$$E_{ji,k} = x_{ji,k}^C (w_i L_i) + \sum_s x_{ji,ks}^M Y_{i,s}.$$

Thus, total revenue of sector k of market j is

$$Y_{j,k} = \sum_i E_{ji,k},$$

and, therefore,

$$Y_{j,k} = \sum_i x_{ji,k} \left[\xi_{i,k}(w_i L_i) + \sum_s \theta_{i,ks}(1 - \varpi_{i,s}) Y_{i,s} \right]. \quad (\text{C.48})$$

The labor market clearing condition is

$$w_j L_j = \sum_k \varpi_{j,k} Y_{j,k}. \quad (\text{C.49})$$

Equilibrium. We use the production cost in (C.44) and the labor supply equation in (1) to write the price indices in (C.45) and (C.47) as

$$P_j = \prod_s \left[\sum_{o:s \in \mathcal{S}_o} \left(\tau_{oj,s} \frac{(w_o)^{\varpi_{i,s}} (P_{o,s}^M)^{1-\varpi_{i,s}}}{\Psi_o(\Phi(\mathbf{w}, \mathbf{P}))} \right)^{-\epsilon_s} \right]^{\frac{\xi_{j,s}}{-\epsilon_s}}, \quad (\text{C.50})$$

$$P_{i,s}^M = \prod_k \left[\sum_o \left(\tau_{oi,k} \frac{(w_o)^{\varpi_{o,k}} (P_{o,k}^M)^{1-\varpi_{o,k}}}{\Psi_o(\Phi(\mathbf{w}, \mathbf{P}))} \right)^{-\epsilon_k} \right]^{\frac{\theta_{i,ks}}{-\epsilon_k}}. \quad (\text{C.51})$$

We rewrite the sectoral market clearing condition in (C.48) using the production cost in (C.44), the gravity sectoral demand in (C.46), and the labor supply equation in (1):

$$Y_{j,k} = \sum_i \frac{\left(\tau_{ji,k} \frac{(w_j)^{\varpi_{j,k}} (P_{j,s}^M)^{1-\varpi_{j,k}}}{\Psi_j(\Phi(\mathbf{w}, \mathbf{P}))} \right)^{-\epsilon_k}}{\sum_o \left(\tau_{oi,k} \frac{(w_o)^{\varpi_{o,k}} (P_{o,k}^M)^{1-\varpi_{o,k}}}{\Psi_o(\Phi(\mathbf{w}, \mathbf{P}))} \right)^{-\epsilon_k}} \left[\xi_{i,k}(w_i \Phi_i(\mathbf{w}, \mathbf{P})) + \sum_s \theta_{i,ks}(1 - \varpi_{i,s}) Y_{i,s} \right]. \quad (\text{C.52})$$

Finally, we use the labor supply equation in (1) to write the labor market clearing condition:

$$w_j \Phi_j(\mathbf{w}, \mathbf{P}) = \sum_k \varpi_{j,k} Y_{j,k}. \quad (\text{C.53})$$

The equilibrium is defined as vectors of wage rates $\mathbf{w} \equiv \{w_i\}_i$, consumption price indices $\mathbf{P} = \{P_j\}_j$, sectoral input price indices $\mathbf{P}^M = \{P_{j,k}\}_{jk}$, and sectoral revenues $\mathbf{Y} = \{Y_{j,k}\}_{jk}$ satisfying equations (C.50)–(C.53) for a given numeraire wage $w_m \equiv 1$.

C.5.2 Counterfactual analysis

We now present the main implications of allowing for non-labor inputs in production. All derivations and matrix definitions are presented below. We obtain a similar system determining relative wage changes across markets:

$$\bar{\gamma}\hat{\omega} = \hat{\eta}. \quad (\text{C.54})$$

However, in this case, the shift in excess labor demand has the following form

$$\hat{\eta} \equiv \check{\alpha}^R \hat{\eta}^R - \bar{\alpha}^C \bar{\phi}^p \hat{\eta}^C + \check{\alpha}^M \hat{\eta}^M.$$

Relative to the baseline model, there are three main differences in the excess labor demand shift. First, revenue shift is defined at the market-sector level:

$$\hat{\eta}_{j,k}^R \equiv -\epsilon_k \sum_i y_{ji,k} \left(\hat{\tau}_{ji,k} + \sum_o x_{oi,k} \hat{\tau}_{oi,k} \right).$$

This is because sectors within the same region have a different composition of intermediate inputs and thus a different production structure. The matrix $\check{\alpha}^R$ simply weights the sector-level shifts using the Leontief inverse matrix that translates revenue shocks in on sector to shocks for its suppliers.

Second, the consumption cost shift is the same as in Proposition 2, since the trade demand for final consumption is the same as in the baseline. The labor supply multiplier $\bar{\alpha}^C$, however, takes into account also for connection across sectors arising from production linkages.

Third, there is a shift in intermediate input cost for each sector-market:

$$\hat{\eta}_{i,s}^M \equiv \sum_{o,k} \theta_{i,ks} x_{oi,k} \hat{\tau}_{oi,k}. \quad (\text{C.55})$$

The matrix $\bar{\alpha}^M$ is the demand multiplier implied by changes in the cost of intermediate inputs through the production network.

C.5.3 Derivation of equation (C.54)

We use bold variables with tilde to denote stacked vectors of sector-market variables, $\tilde{\mathbf{y}} \equiv [y_{i,s}]_{is}$, bold variables with two dots to denote matrices with sector-market to sector-market variables, $\check{\mathbf{y}} \equiv [y_{ij,ks}]_{is,ks}$, and bold variables with checks to denote matrices with market to sector-market variables $\check{\mathbf{y}} \equiv [y_{ij,k}]_{ik,j}$.

The production cost equation in (C.44) implies that

$$\hat{p}_{i,s} = \hat{w}_i \varpi_{i,s} + \hat{P}_{i,s}^M (1 - \varpi_{i,s}). \quad (\text{C.56})$$

Using this expression and the input price index in (C.45), we have that

$$\hat{P}_{i,s}^M = \hat{\eta}_{i,s}^M + \sum_{o,k} \theta_{i,ks} x_{oi,k} \left[\hat{w}_o \varpi_{o,k} + \hat{P}_{o,k}^M (1 - \varpi_{o,k}) - \sum_j \psi_{oj} \hat{L}_j \right]$$

where

$$\hat{\eta}_{i,s}^M \equiv \sum_{o,k} \theta_{i,ks} x_{oi,k} \hat{\tau}_{oi,k}. \quad (\text{C.57})$$

In matrix form and invert it as

$$\hat{\mathbf{P}}^M = \hat{\boldsymbol{\eta}}^M + \check{\mathbf{x}}^M \left(\check{\boldsymbol{\omega}} \hat{\boldsymbol{\omega}} + (\check{\mathbf{I}} - \check{\boldsymbol{\omega}}) \hat{\mathbf{P}}^M - \check{\mathbf{i}} \bar{\boldsymbol{\psi}} \hat{\mathbf{L}} \right)$$

where $\check{\mathbf{x}}^M \equiv [\theta_{i,ks} x_{oi,k}]_{is,ok}$. We define $\check{\boldsymbol{\omega}}$ as a $NS \times N$ matrix with row of sector k in market o given by $\varpi_{o,k}$ in the column o and zero in other columns, and $\check{\mathbf{i}}$ as a $NS \times N$ matrix whose rows of sector s in

market o has a one in the column of market o and zero in other columns.

Thus,

$$\hat{\mathbf{P}}^M = \ddot{\boldsymbol{\lambda}} \left(\hat{\boldsymbol{\eta}}^M + \ddot{\mathbf{x}}^M \left(\ddot{\boldsymbol{\omega}} \hat{\boldsymbol{w}} - \check{\boldsymbol{\psi}} \hat{\mathbf{L}} \right) \right) \quad (\text{C.58})$$

where $\ddot{\boldsymbol{\lambda}} \equiv \left(\ddot{\mathbf{I}} - \ddot{\mathbf{x}}^M (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \right)^{-1}$.

The consumption price index change is similar to what we had before:

$$\hat{P}_i = \hat{\eta}_i^C + \sum_{o,k} \xi_{i,k} x_{oi,k} \left[\hat{w}_o \varpi_{o,k} + \hat{P}_{o,k}^M (1 - \varpi_{o,k}) - \sum_j \psi_{oj} \hat{L}_j \right]$$

where

$$\hat{\eta}_i^C \equiv \sum_{o,k} \xi_{i,k} x_{oi,k} \hat{\tau}_{oi,k}. \quad (\text{C.59})$$

Define the $N \times SN$ matrix $\ddot{\mathbf{x}}^{C'} \equiv [\xi_{i,k} x_{oi,k}]_{i,ok}$. The price index equation above can be written in matrix form:

$$\hat{\mathbf{P}} = \hat{\boldsymbol{\eta}}^C + \ddot{\mathbf{x}}^{C'} \left(\ddot{\boldsymbol{\omega}} \hat{\boldsymbol{w}} + (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \hat{\mathbf{P}}^M - \check{\boldsymbol{\psi}} \hat{\mathbf{L}} \right).$$

By applying equation (C.58) into this expression,

$$\hat{\mathbf{P}} = \hat{\boldsymbol{\eta}}^C + \ddot{\mathbf{x}}^{C'} (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \hat{\boldsymbol{\eta}}^M + \ddot{\mathbf{x}}^{C'} \left((\ddot{\mathbf{I}} + (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \ddot{\mathbf{x}}^M) \ddot{\boldsymbol{\omega}} \hat{\boldsymbol{w}} - (\ddot{\mathbf{I}} + (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \ddot{\mathbf{x}}^M) \check{\boldsymbol{\psi}} \hat{\mathbf{L}} \right). \quad (\text{C.60})$$

From the labor supply equation in (1),

$$\hat{\mathbf{L}} = \bar{\boldsymbol{\phi}}^w \hat{\boldsymbol{w}} + \bar{\boldsymbol{\phi}}^p \hat{\mathbf{P}}.$$

Using (C.60),

$$\hat{\mathbf{L}} = \bar{\boldsymbol{\phi}}^w \hat{\boldsymbol{w}} + \bar{\boldsymbol{\phi}}^p \hat{\boldsymbol{\eta}}^C + \bar{\boldsymbol{\phi}}^p \ddot{\mathbf{x}}^{C'} (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \hat{\boldsymbol{\eta}}^M + \bar{\boldsymbol{\phi}}^p \ddot{\mathbf{x}}^{C'} \left((\ddot{\mathbf{I}} + (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \ddot{\mathbf{x}}^M) \ddot{\boldsymbol{\omega}} \hat{\boldsymbol{w}} - (\ddot{\mathbf{I}} + (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \ddot{\mathbf{x}}^M) \check{\boldsymbol{\psi}} \hat{\mathbf{L}} \right)$$

Thus,

$$\hat{\mathbf{L}} = \bar{\boldsymbol{\varphi}}^w \hat{\boldsymbol{w}} + \bar{\boldsymbol{\phi}}^p \left(\hat{\boldsymbol{\eta}}^C + \ddot{\mathbf{x}}^{C'} (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \hat{\boldsymbol{\eta}}^M \right) \quad (\text{C.61})$$

where

$$\begin{aligned} \bar{\boldsymbol{\varphi}}^w &\equiv \left(\bar{\boldsymbol{\phi}}^w + \bar{\boldsymbol{\phi}}^p \ddot{\mathbf{x}}^{C'} \left(\ddot{\mathbf{I}} + (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \ddot{\mathbf{x}}^M \right) \ddot{\boldsymbol{\omega}} \right) \\ \bar{\boldsymbol{\rho}} &\equiv \left(\bar{\mathbf{I}} + \bar{\boldsymbol{\phi}}^p \ddot{\mathbf{x}}^{C'} \left(\ddot{\mathbf{I}} + (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \ddot{\mathbf{x}}^M \right) \check{\boldsymbol{\psi}} \right)^{-1}. \end{aligned}$$

Together, expressions (C.56), (C.58) and (C.61) imply that

$$\begin{aligned} \hat{\mathbf{p}} &= \left[\left(\ddot{\mathbf{I}} + (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \ddot{\mathbf{x}}^M \right) \ddot{\boldsymbol{\omega}} - (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \ddot{\mathbf{x}}^M \check{\boldsymbol{\psi}} \bar{\boldsymbol{\varphi}}^w \right] \hat{\boldsymbol{w}} \\ &+ (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \left[\ddot{\mathbf{x}}^M \check{\boldsymbol{\psi}} \bar{\boldsymbol{\phi}}^p \hat{\boldsymbol{\eta}}^C + \left(\ddot{\mathbf{I}} + \ddot{\mathbf{x}}^M \check{\boldsymbol{\psi}} \bar{\boldsymbol{\phi}}^p \ddot{\mathbf{x}}^{C'} (\ddot{\mathbf{I}} - \ddot{\boldsymbol{\omega}}) \ddot{\boldsymbol{\lambda}} \right) \hat{\boldsymbol{\eta}}^M \right]. \end{aligned} \quad (\text{C.62})$$

From equation (C.48),

$$\hat{Y}_{j,k} = \sum_i \frac{E_{ji,k}}{Y_{j,k}} \left[\hat{x}_{ji,k} + \frac{x_{ji,k}^C w_i L_i}{E_{ji,k}} (\hat{w}_i + \hat{L}_i) + \sum_s \frac{x_{ji,ks}^M Y_{i,s}}{E_{ji,k}} \hat{Y}_{i,s} \right],$$

which implies that

$$\hat{Y}_{j,k} = \sum_i y_{ji,k} \left[\hat{x}_{ji,k} + \pi_{ji,k} (\hat{w}_i + \hat{L}_i) + (1 - \pi_{ji,k}) \left(\sum_s \mu_{ji,ks} \hat{Y}_{i,s} \right) \right]$$

where $y_{ji,k} \equiv \frac{E_{ji,k}}{Y_{j,k}}$ is the revenue share of market i from sales to market j in sector k ; $\pi_{ji,k} \equiv \frac{x_{ji,k}^C (w_i L_i)}{E_{ji,k}}$ is the fraction of final consumption in total spending of market i on sector k of market j ; $\mu_{ji,ks} \equiv \frac{x_{ji,ks}^M Y_{i,s}}{\sum_s x_{ji,ks}^M Y_{i,s}}$ is the fraction of intermediate inputs purchases of sector s in all intermediate input flows from sector k of market j to market i .

From equation (C.46),

$$\hat{x}_{ji,k} = -\epsilon_k \left(\hat{\tau}_{ji,k} + \hat{p}_{j,k} - \sum_d \psi_{jd} \hat{L}_d \right) + \sum_o x_{oi,k} \epsilon_k \left(\hat{\tau}_{oi,k} + \hat{p}_{o,k} - \sum_d \psi_{od} \hat{L}_d \right).$$

Let us define the revenue shift of sector k of market i as

$$\hat{\eta}_{j,k}^R \equiv -\epsilon_k \sum_i y_{ji,k} \left(\hat{\tau}_{ji,k} + \sum_o x_{oi,k} \hat{\tau}_{oi,k} \right)$$

and the revenue elasticity of sector k of market j to changes in the production cost of sector k in market o as

$$\chi_{oj,k} = \sum_i y_{ji,k} (x_{oi,k} \epsilon_k - \mathbb{I}_{[i=j]}) \epsilon_k.$$

Applying these definitions to the equations above, we get that

$$\hat{Y}_{j,k} = \hat{\eta}_{j,k}^R + \sum_o \chi_{oj,k} \left(\hat{p}_{o,k} - \sum_d \psi_{od} \hat{L}_d \right) + \sum_i y_{ji,k} \pi_{ji,k} (\hat{w}_i + \hat{L}_i) + \sum_s \sum_i y_{ji,k} (1 - \pi_{ji,k}) \mu_{ji,ks} \hat{Y}_{i,s}.$$

We can then write this system in matrix form:

$$\hat{\mathbf{Y}} = \hat{\boldsymbol{\eta}}^R + \check{\boldsymbol{\chi}} \left(\hat{\mathbf{p}} - \check{\mathbf{i}} \bar{\boldsymbol{\psi}} \hat{\mathbf{L}} \right) + \check{\boldsymbol{\pi}} \left(\hat{\mathbf{w}} + \hat{\mathbf{L}} \right) + \check{\boldsymbol{\mu}} \hat{\mathbf{Y}},$$

where $\check{\boldsymbol{\chi}}$ is the $SN \times SN$ matrix with row of market j sector k given by $\chi_{oj,k}$ for the columns of markets o in sector k and zeros for the columns of other sectors; $\check{\boldsymbol{\mu}} \equiv [y_{ji,k} (1 - \pi_{ji,k}) \mu_{ji,ks}]_{jk, is}$ is the $SN \times SN$ matrix with input revenue shares; and $\check{\boldsymbol{\pi}}$ is the $SN \times N$ matrix with row of market j sector k given by $y_{ji,k} \pi_{ji,k}$ for the columns of market i .

Thus,

$$\hat{\mathbf{Y}} = \left(\mathbf{I} - \check{\boldsymbol{\mu}} \right)^{-1} \left(\hat{\boldsymbol{\eta}}^R + \check{\boldsymbol{\chi}} \left(\hat{\mathbf{p}} - \check{\mathbf{i}} \bar{\boldsymbol{\psi}} \hat{\mathbf{L}} \right) + \check{\boldsymbol{\pi}} \left(\hat{\mathbf{w}} + \hat{\mathbf{L}} \right) \right). \quad (\text{C.63})$$

Finally, we need to solve for the labor market clearing condition. Define $l_{jk} \equiv \frac{\varpi_{j,k} Y_{j,k}}{w_j L_j}$ as the share of labor income coming from sector k in market j . From C.53,

$$\hat{w}_j + \hat{L}_j = \sum_k l_{j,k} \hat{Y}_{j,k},$$

which implies that

$$\hat{\mathbf{w}} + \hat{\mathbf{L}} = \check{\boldsymbol{\ell}}' \hat{\mathbf{Y}}$$

where $\check{\ell}'$ is the $N \times NS$ matrix with row j with entries $\ell_{j,k}$ in the columns of market j for different sectors k and zeros in the columns of other markets. By replacing equation (C.63) into the expression above, we have

$$\hat{w} + \hat{L} = \check{\ell}' \left(\check{I} - \check{\mu} \right)^{-1} \left(\hat{\eta}^R + \check{\chi} \left(\hat{p} - \check{i}\bar{\psi}\hat{L} \right) + \check{\pi} \left(\hat{w} + \hat{L} \right) \right),$$

so that

$$\left(\bar{I} - \check{\ell}' \left(\check{I} - \check{\mu} \right)^{-1} \check{\pi} \right) \hat{w} + \left(\bar{I} - \check{\ell}' \left(\check{I} - \check{\mu} \right)^{-1} \left(\check{\pi} - \check{\chi}\check{i}\bar{\psi} \right) \right) \hat{L} = \check{\ell}' \left(\check{I} - \check{\mu} \right)^{-1} \left(\hat{\eta}^R + \check{\chi}\hat{p} \right).$$

Substituting expressions (C.61) and (C.62) into this expression,

$$\bar{\gamma}\hat{w} = \hat{\eta}$$

where

$$\begin{aligned} \bar{\gamma} &\equiv \bar{I} + \left(\bar{I} - \check{\ell}' \left(\check{I} - \check{\mu} \right)^{-1} \left(\check{\pi} - \check{\chi}\check{i}\bar{\psi} \right) \right) \bar{\varphi}^w \\ &- \check{\ell}' \left(\check{I} - \check{\mu} \right)^{-1} \left(\check{\pi} + \check{\chi} \left[\left(\check{I} + \left(\check{I} - \check{\omega} \right) \check{\lambda}\check{x}^M \right) \check{\omega} - \left(\check{I} - \check{\omega} \right) \check{\lambda}\check{x}^M \check{i}\bar{\psi}\bar{\varphi}^w \right] \right) \\ \hat{\eta} &\equiv \check{\alpha}^R \hat{\eta}^R - \check{\alpha}^C \bar{\varphi}^p \hat{\eta}^C + \check{\alpha}^{M'} \hat{\eta}^M \end{aligned}$$

and

$$\begin{aligned} \check{\alpha}^R &= \check{\ell}' \left(\check{I} - \check{\mu} \right)^{-1} \\ \check{\alpha}^C &= \bar{I} - \check{\ell}' \left(\check{I} - \check{\mu} \right)^{-1} \left[\check{\pi} - \check{\chi} \left(\check{I} - \left(\check{I} - \check{\omega} \right) \check{\lambda}\check{x}^M \right) \check{i}\bar{\psi} \right] \\ \check{\alpha}^{M'} &= \check{\ell}' \left(\check{I} - \check{\mu} \right)^{-1} \check{\chi} \left(\check{I} - \check{\omega} \right) \check{\lambda} \left(\check{I} + \check{x}^M \check{i}\bar{\psi}\bar{\varphi}^p \check{x}^{C'} \left(\check{I} - \check{\omega} \right) \check{\lambda} \right) \\ &- \left(\bar{I} - \check{\ell}' \left(\check{I} - \check{\mu} \right)^{-1} \left(\check{\pi} - \check{\chi}\check{i}\bar{\psi} \right) \right) \bar{\varphi}^p \check{x}^{C'} \left(\check{I} - \check{\omega} \right) \check{\lambda}. \end{aligned}$$

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